



UNIVERSITY OF  
CALGARY

Department of Computer Science

# Compressive Volume Rendering

**Xiaoyang Liu**  
*MSc Student*

**Usman R. Alim**  
*Assistant Professor*



**VISAGG**

Visualization and Graphics Group

## 1. Motivation

## 2. Research Question

## 3. Methodologies

## 4. Results

## 5. Conclusion

High Density Displays ▶

Ray-Casting  
for Volume Rendering ▶

High Density Displays ►



Ray-Casting  
for Volume Rendering ►

High Density Displays ►

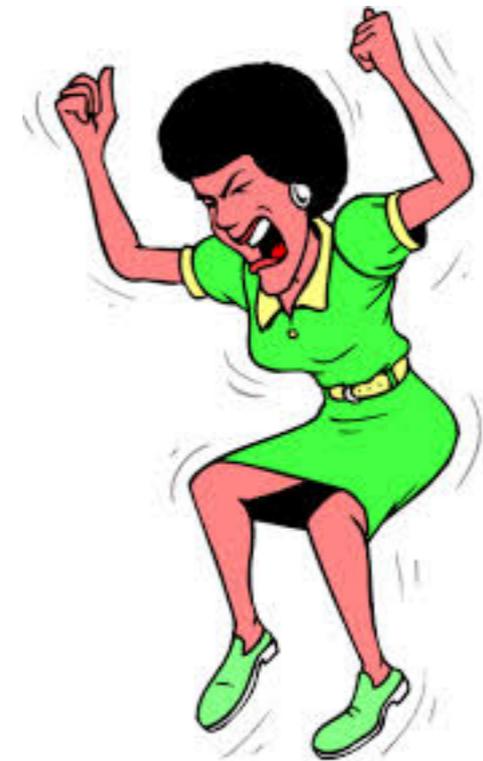


Ray-Casting  
for Volume Rendering ►

**Rendering time is  
proportional to the  
number of pixels**

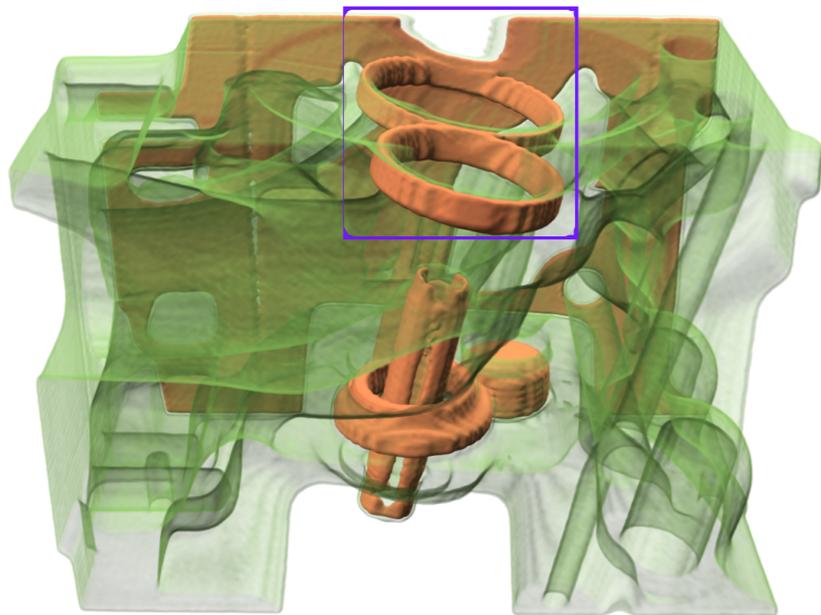


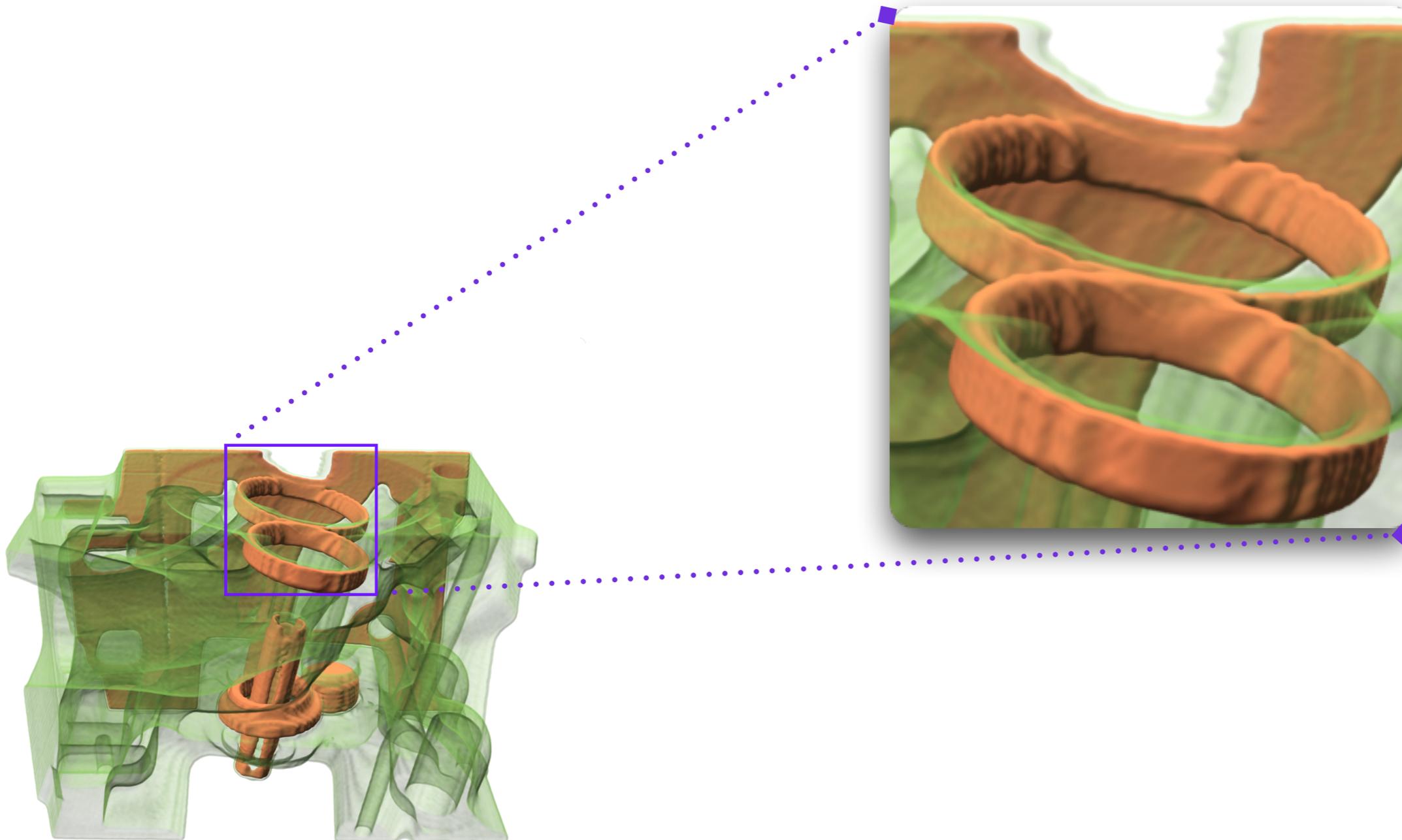
**TIME  
CONSUMING**

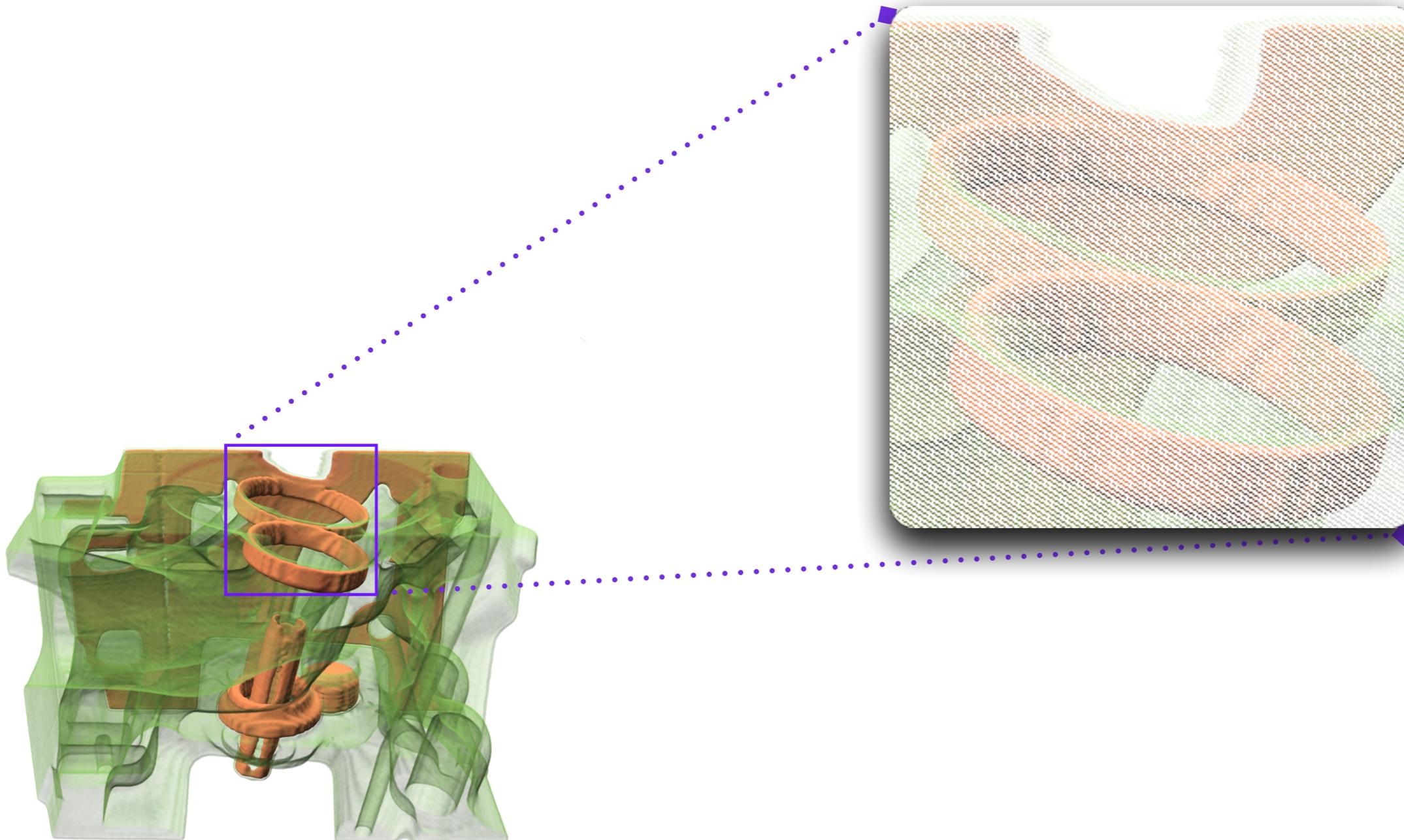


**EFFICIENCY**

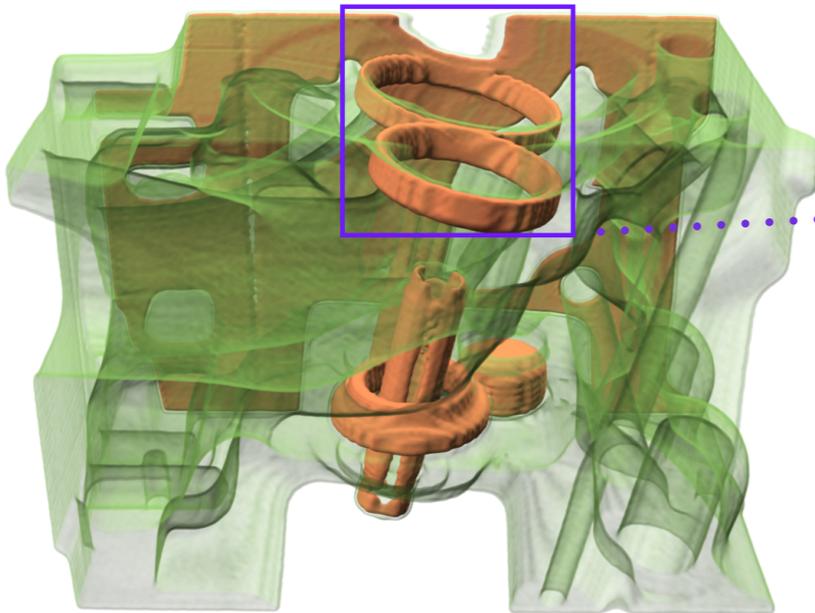
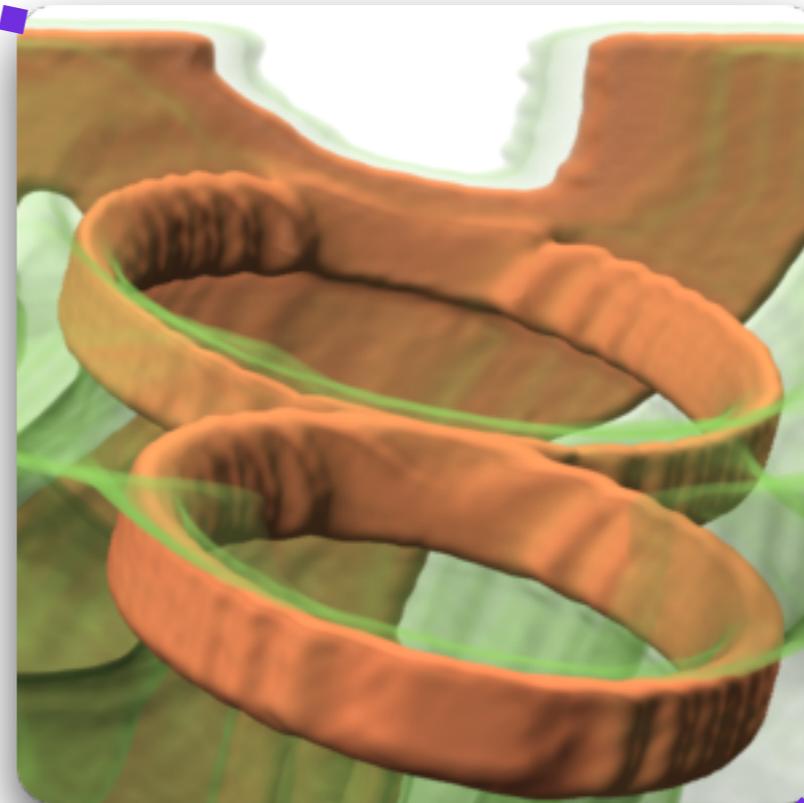
1. Motivation
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How to do this 



With good quality 

1. Motivation
2. Research Question
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## Distribution Algorithms

- Random Distribution
  - *Inhomogeneous regions*
- Low-Discrepancy Distribution
  - *Allow for the progressive update*

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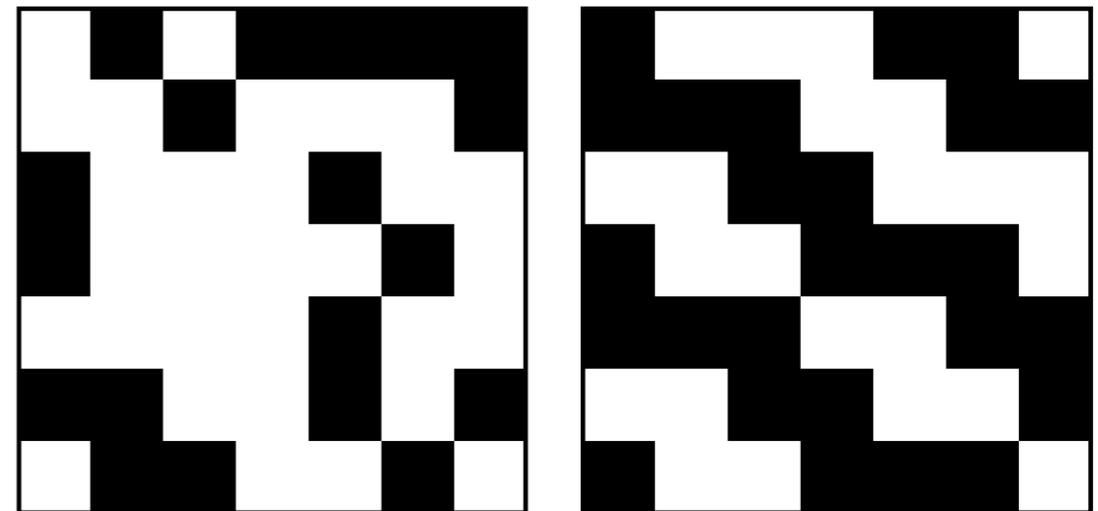
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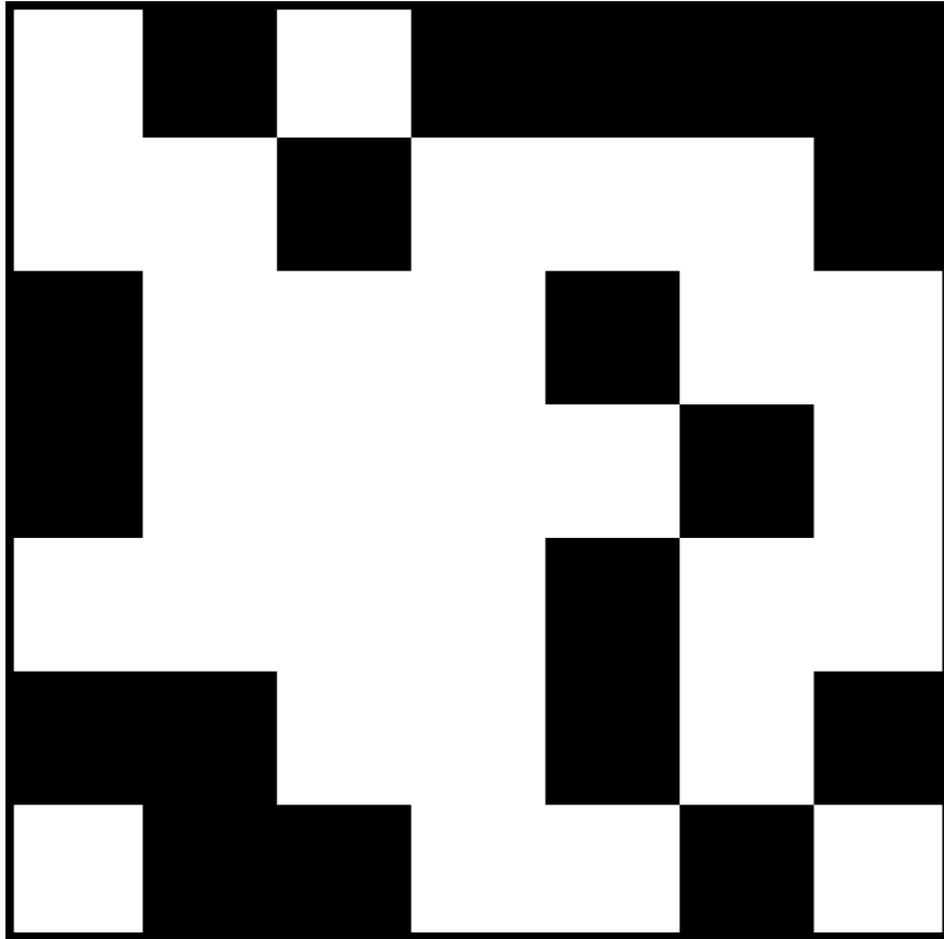
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*Masks for 50% missing pixels*

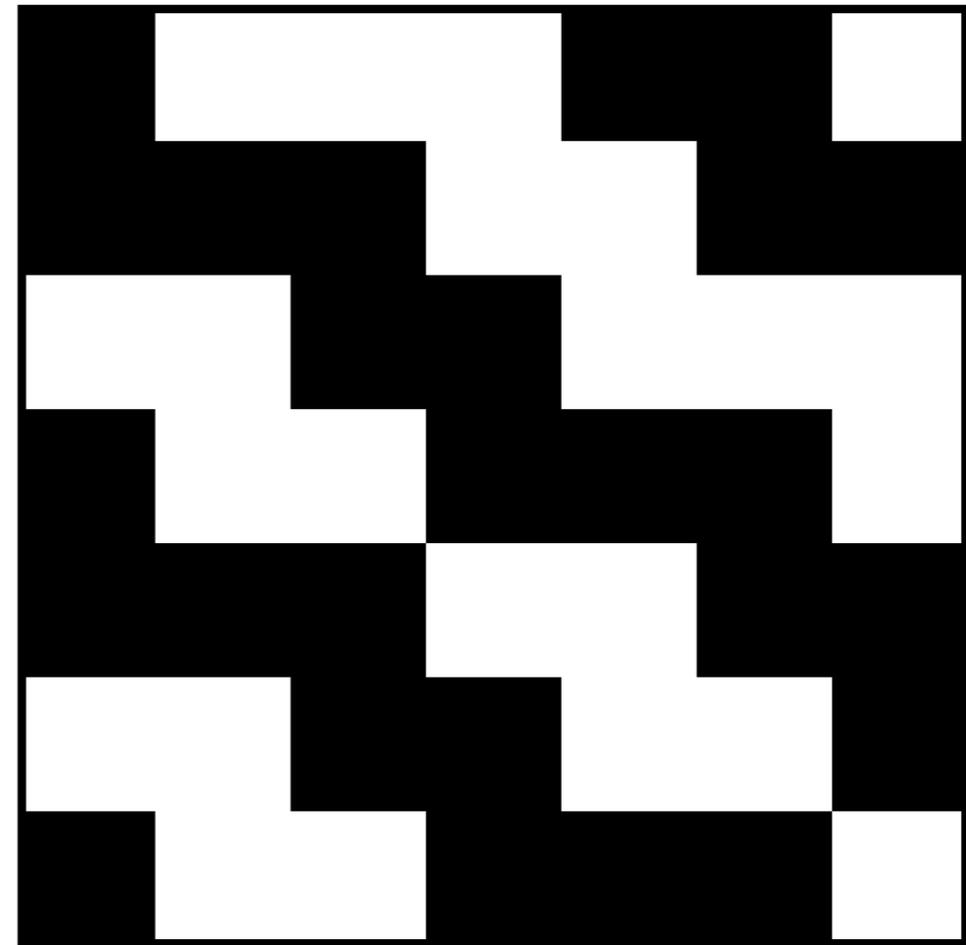


*Random  
Distribution*

*Low-Discrepancy (LD)  
Distribution via  
Pixel Shuffle*



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## CS-Wavelet [Sen and Darabi, 2011, TVCG]

Compressed Sensing ▶

Sensing Mechanism ▶

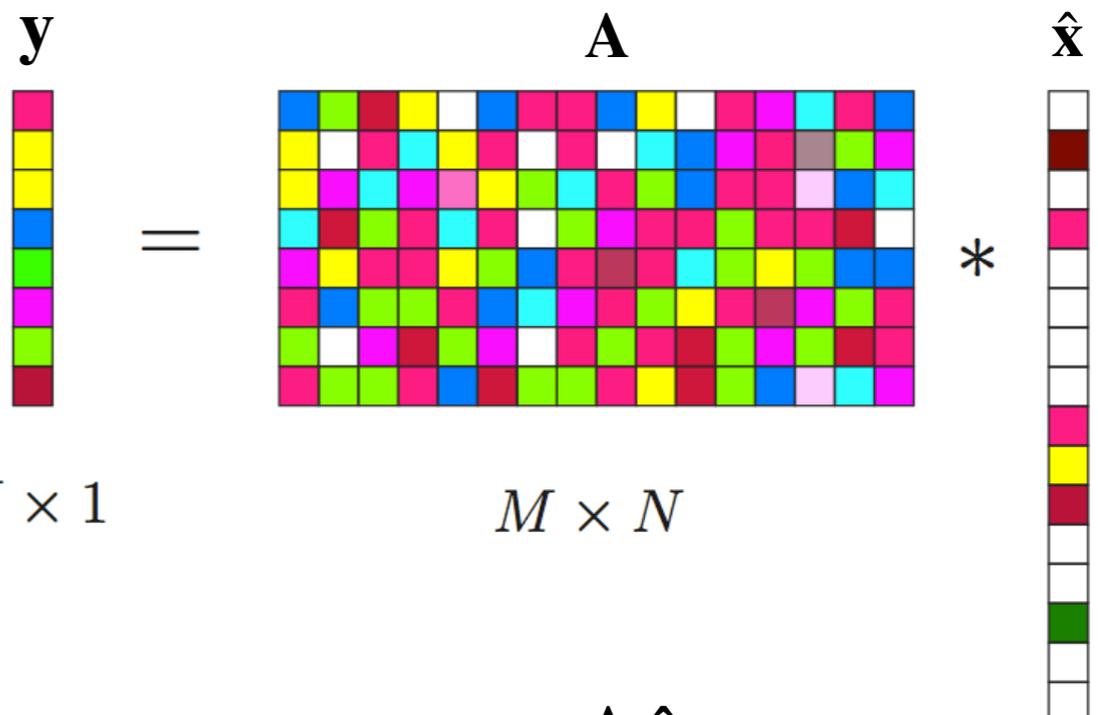
Restricted Isometry  
Condition (RIC) ▶

$$y = A\hat{x}$$

$$y = \underbrace{S\Phi^{-1}W^{-1}}_A \hat{x}_b$$

## CS-Wavelet [Sen and Darabi, 2011, TVCG]

Compressed Sensing ►



$$\begin{matrix} \mathbf{y} \\ M \times 1 \end{matrix} = \begin{matrix} \mathbf{A} \\ M \times N \end{matrix} * \begin{matrix} \hat{\mathbf{x}} \\ N \times 1 \end{matrix}$$

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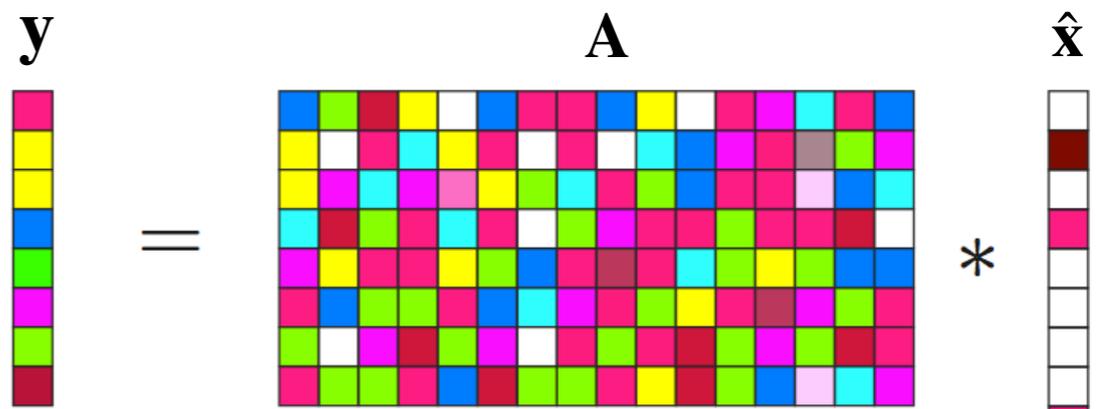
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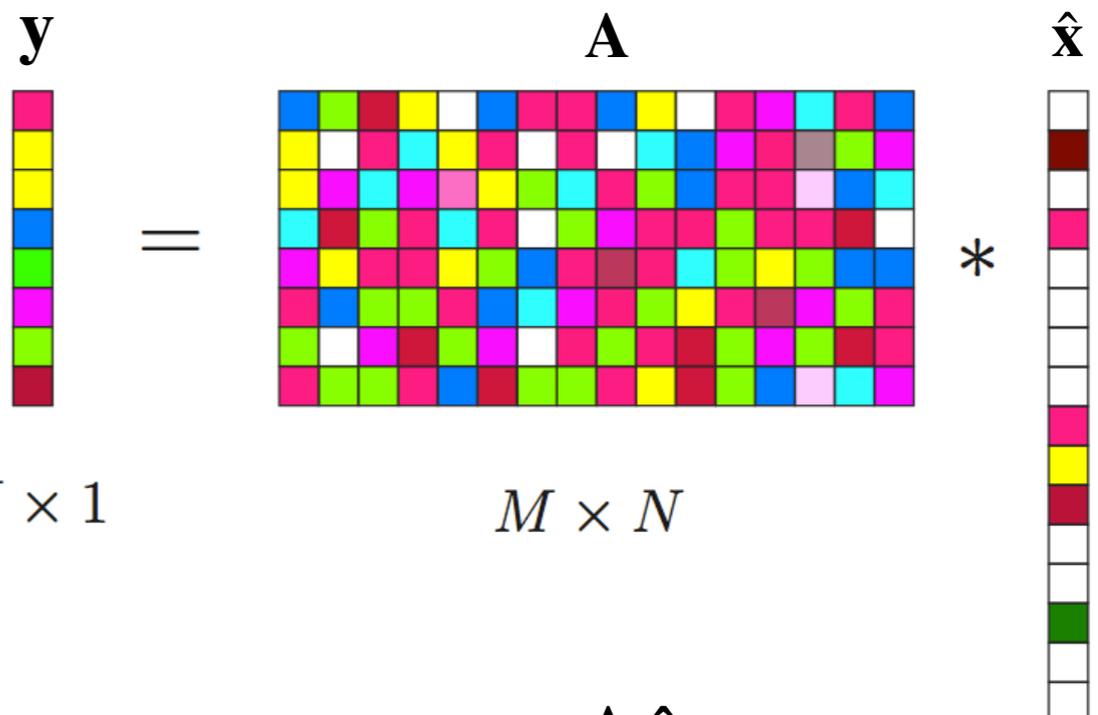
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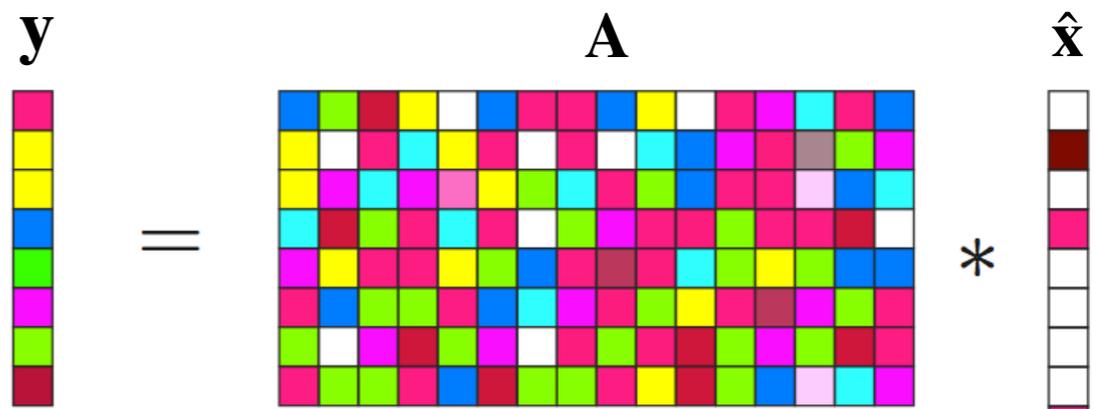
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Incoherence ►

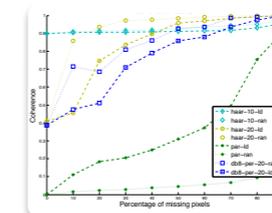
## CS-Wavelet [Sen and Darabi, 2011, TVCG]

Compressed Sensing ►

$$\begin{array}{c} \mathbf{y} \\ \color{red}\square \\ \color{yellow}\square \\ \color{yellow}\square \\ \color{blue}\square \\ \color{green}\square \\ \color{magenta}\square \\ \color{green}\square \\ \color{red}\square \\ M \times 1 \end{array} = \begin{array}{c} \mathbf{A} \\ \color{blue}\square \color{green}\square \color{red}\square \color{yellow}\square \color{white}\square \color{blue}\square \color{magenta}\square \color{yellow}\square \color{white}\square \color{magenta}\square \color{cyan}\square \color{magenta}\square \color{blue}\square \\ \color{yellow}\square \color{white}\square \color{cyan}\square \color{yellow}\square \color{magenta}\square \color{cyan}\square \color{blue}\square \color{magenta}\square \color{cyan}\square \color{magenta}\square \color{cyan}\square \color{magenta}\square \color{cyan}\square \\ \color{cyan}\square \color{red}\square \color{green}\square \color{cyan}\square \color{magenta}\square \color{white}\square \color{green}\square \color{magenta}\square \color{cyan}\square \color{magenta}\square \color{cyan}\square \color{magenta}\square \color{cyan}\square \\ \color{magenta}\square \color{yellow}\square \color{magenta}\square \color{yellow}\square \color{blue}\square \color{magenta}\square \color{cyan}\square \color{green}\square \color{magenta}\square \color{cyan}\square \color{magenta}\square \color{cyan}\square \color{magenta}\square \\ \color{green}\square \color{white}\square \color{magenta}\square \color{red}\square \color{green}\square \color{magenta}\square \color{cyan}\square \color{green}\square \color{magenta}\square \color{cyan}\square \color{magenta}\square \color{cyan}\square \color{magenta}\square \\ \color{magenta}\square \color{green}\square \color{magenta}\square \color{blue}\square \color{red}\square \color{green}\square \color{yellow}\square \color{red}\square \color{green}\square \color{magenta}\square \color{cyan}\square \color{magenta}\square \color{cyan}\square \\ \color{green}\square \color{white}\square \color{magenta}\square \color{red}\square \color{green}\square \color{magenta}\square \color{cyan}\square \color{green}\square \color{magenta}\square \color{cyan}\square \color{magenta}\square \color{cyan}\square \color{magenta}\square \\ \color{magenta}\square \color{green}\square \color{magenta}\square \color{blue}\square \color{red}\square \color{green}\square \color{yellow}\square \color{red}\square \color{green}\square \color{magenta}\square \color{cyan}\square \color{magenta}\square \color{cyan}\square \\ M \times N \end{array} * \begin{array}{c} \hat{\mathbf{x}} \\ \color{brown}\square \\ \color{white}\square \\ \color{magenta}\square \\ \color{white}\square \\ \color{white}\square \\ \color{white}\square \\ \color{white}\square \\ \color{white}\square \\ \color{magenta}\square \\ \color{yellow}\square \\ \color{red}\square \\ \color{white}\square \\ \color{white}\square \\ \color{green}\square \\ \color{white}\square \\ \color{white}\square \\ N \times 1 \end{array}$$

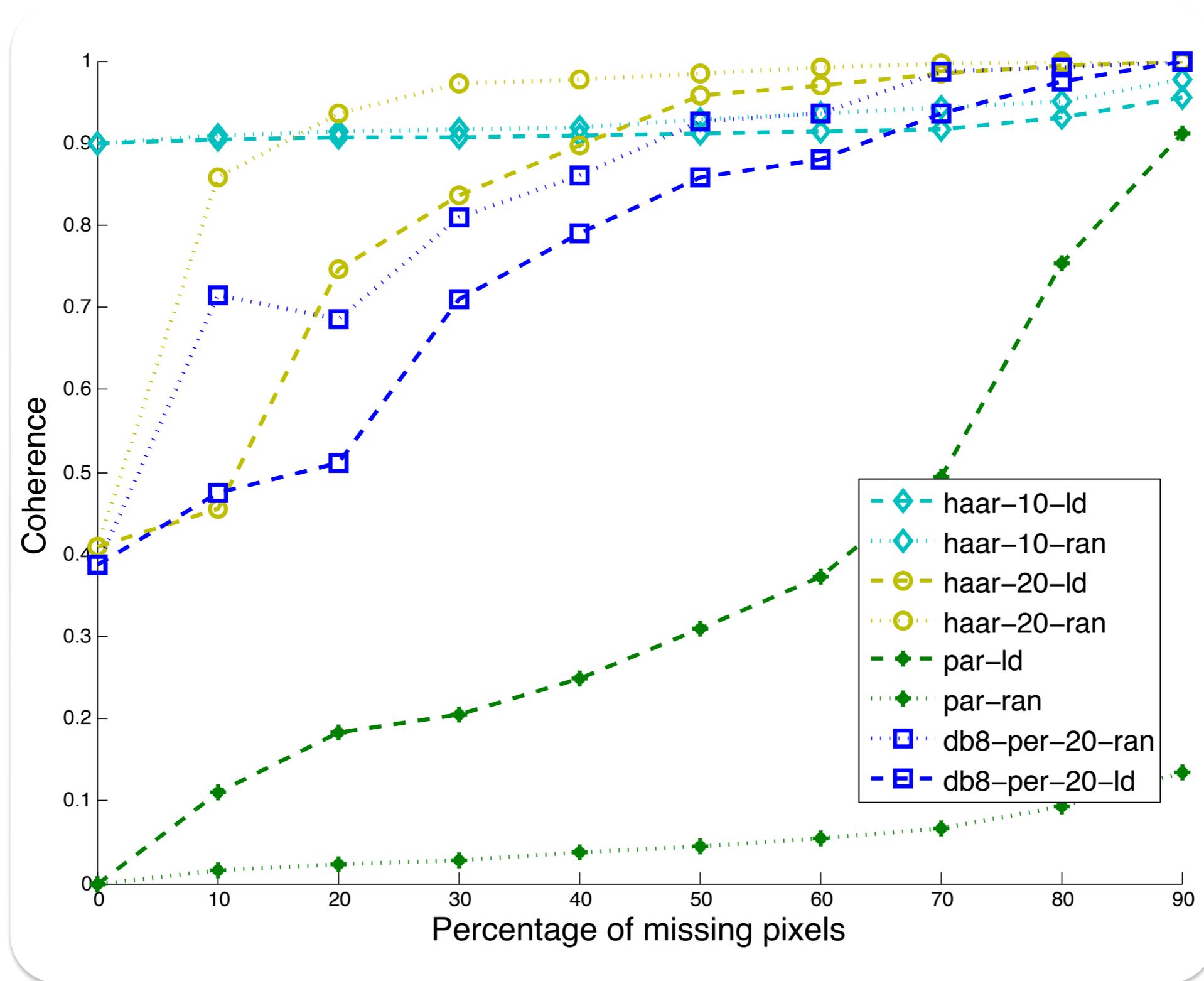
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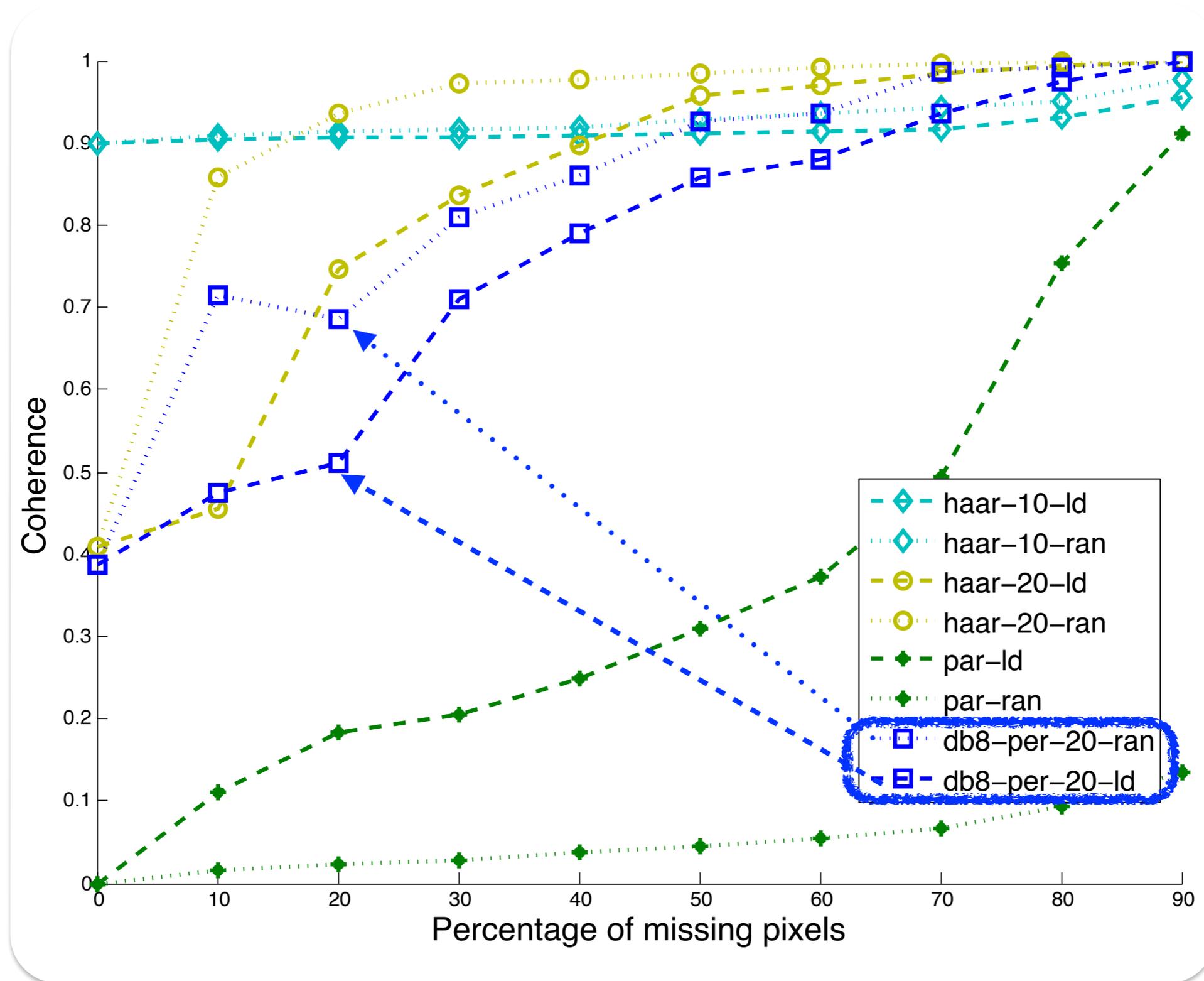
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Sensing Mechanism ►

Incoherence ►





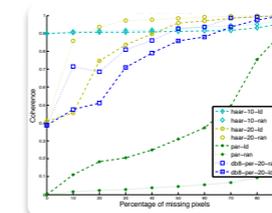
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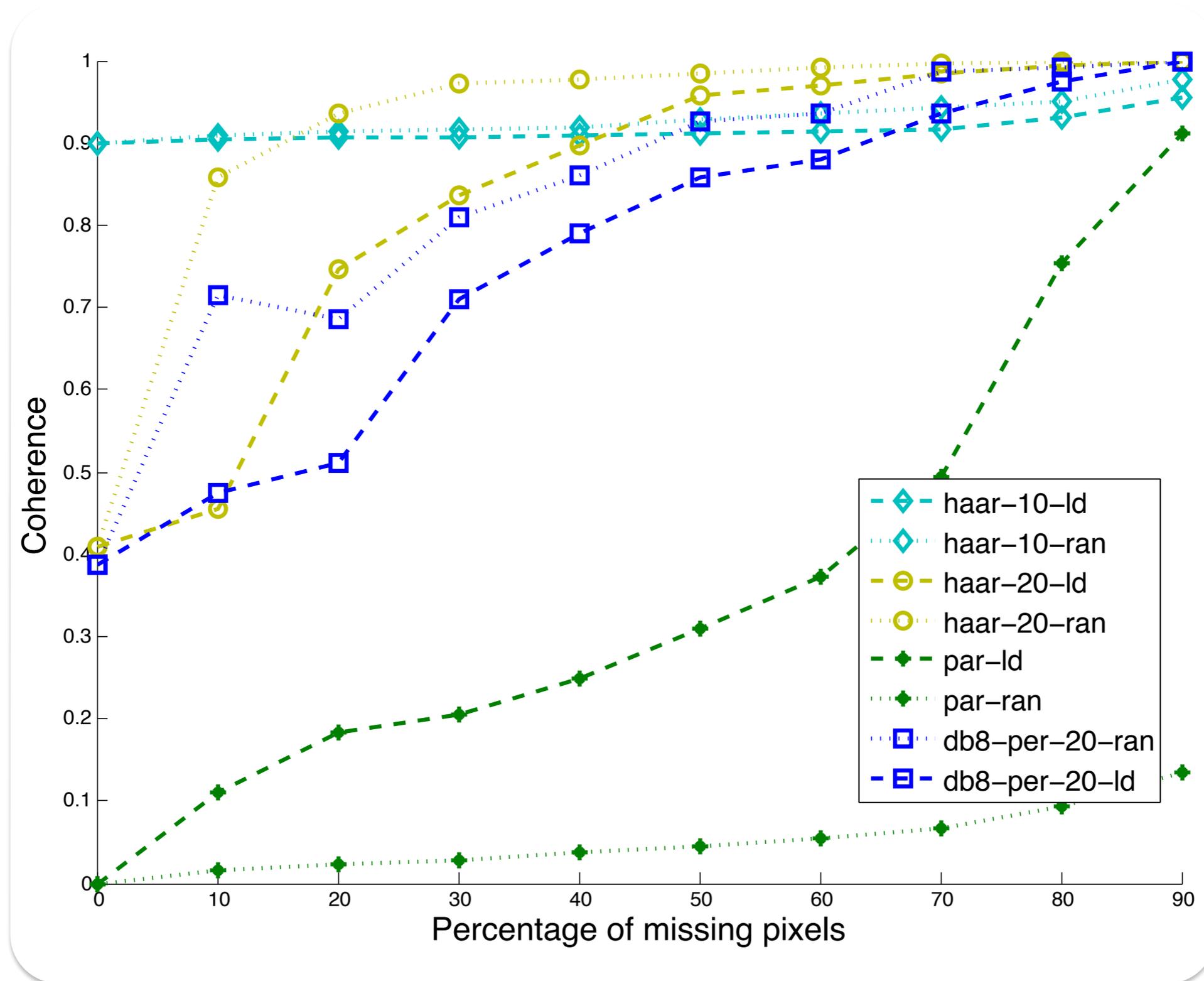
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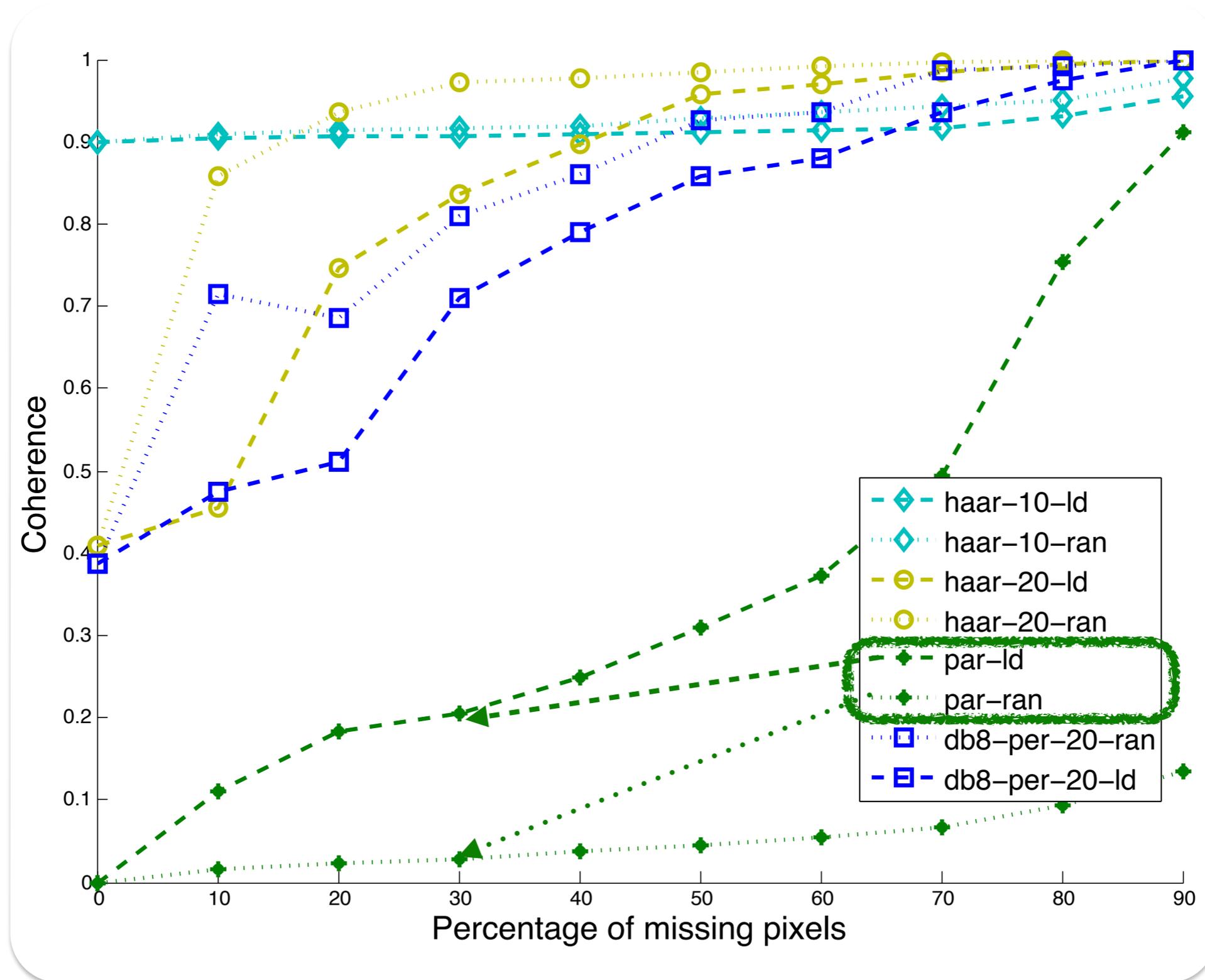
## CS-Gradient

Incoherence ▶

Sparsity in gradient  
domain ▶

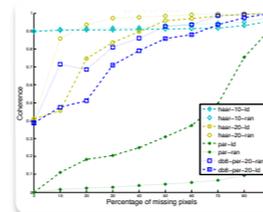






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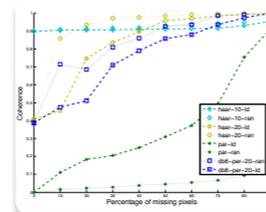
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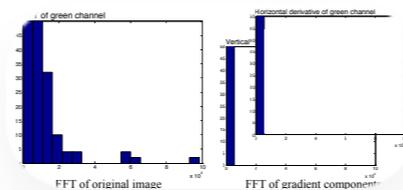
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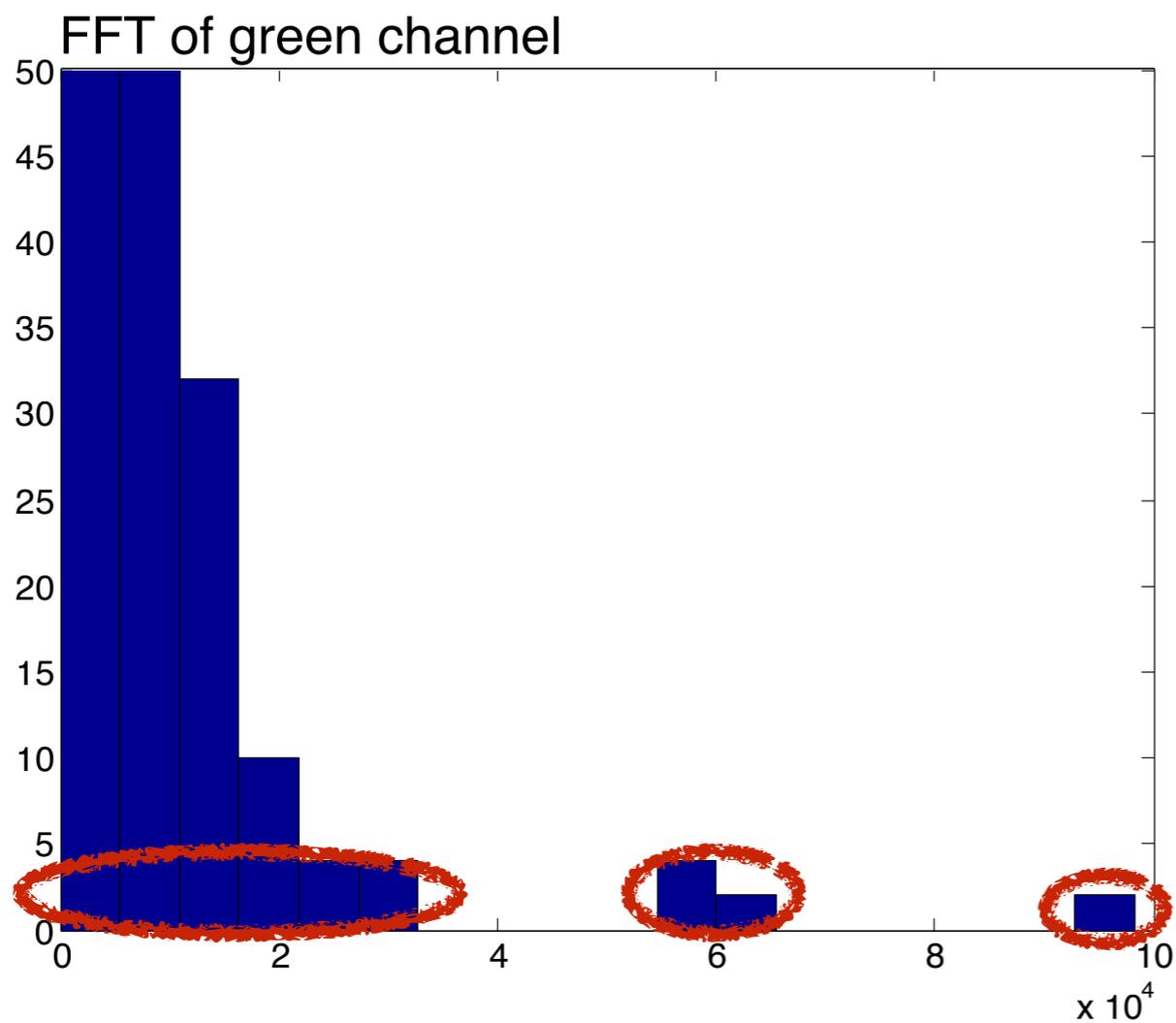
Incoherence ►



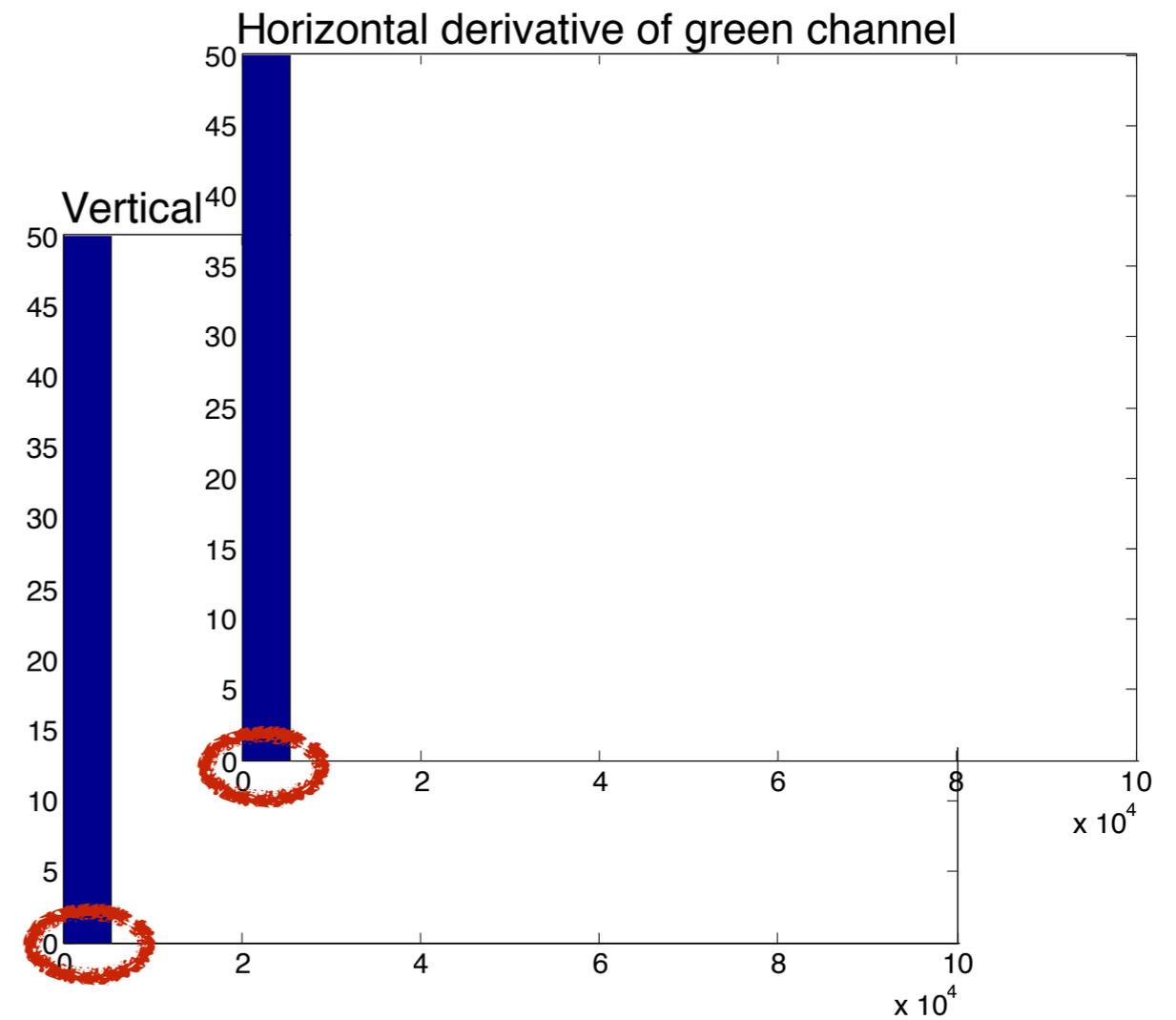
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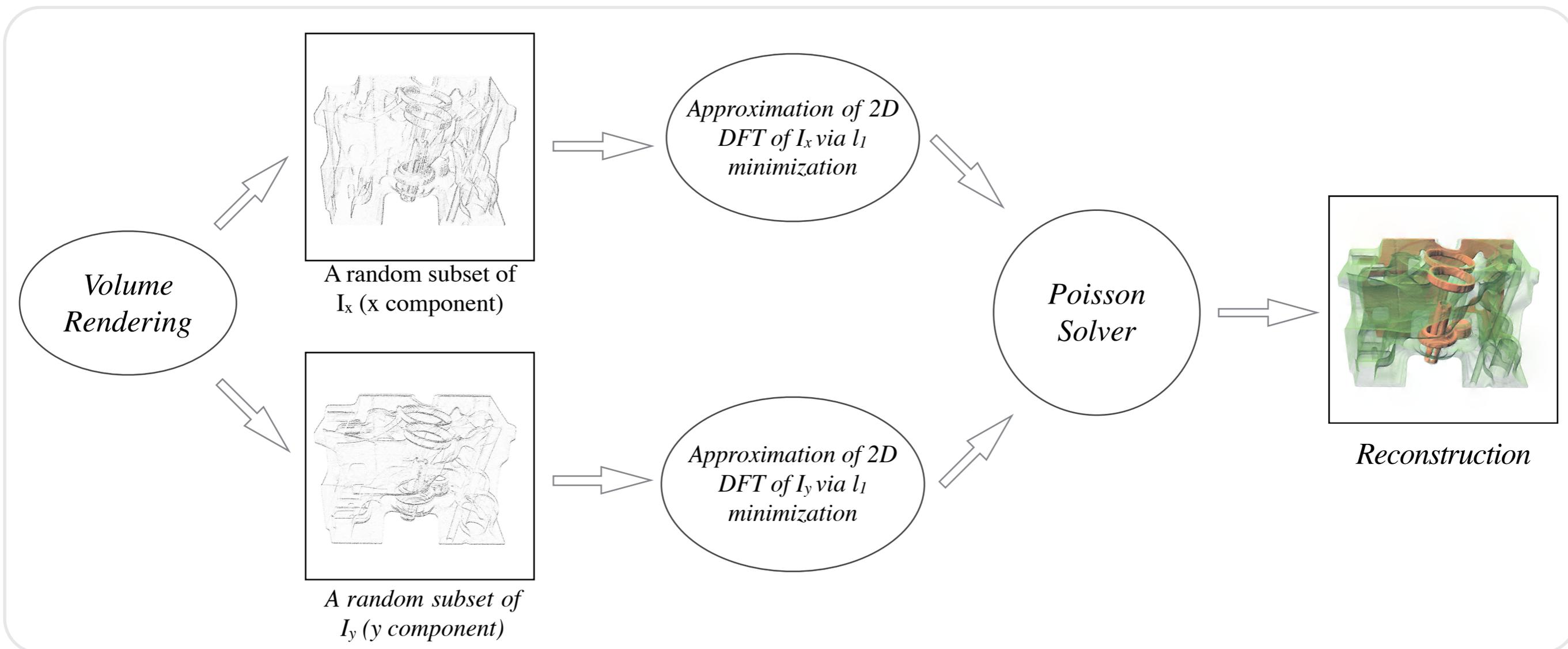
FFT of original image



FFT of gradient components

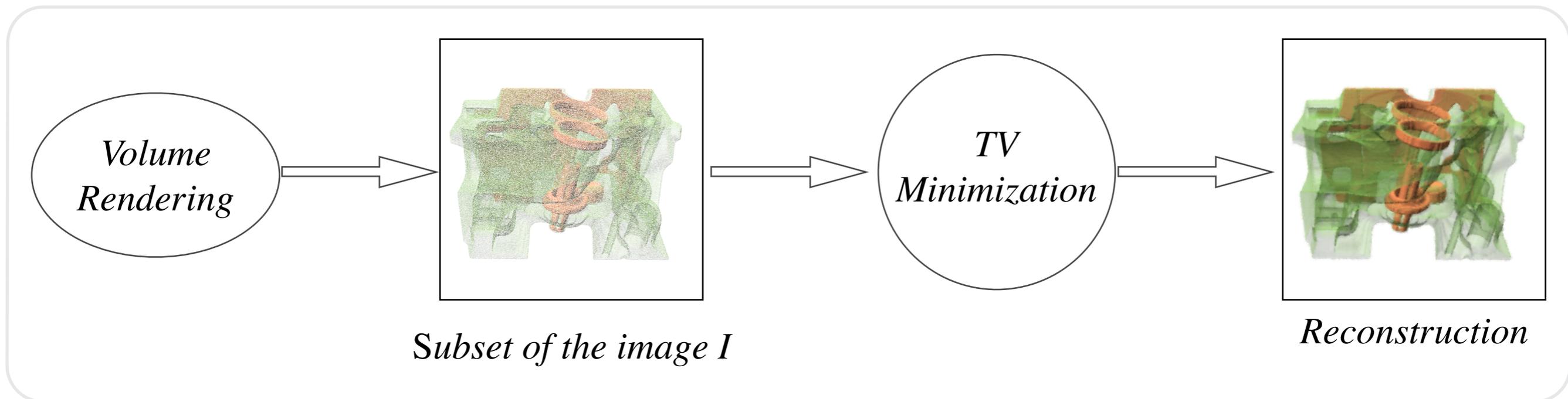
## CS-Gradient

$$\min \|\hat{\mathbf{x}}_1\|_1 \quad \text{subject to} \quad \|\mathbf{S}\mathbf{F}^{-1}\hat{\mathbf{x}}_1 - \mathbf{y}_1\|_2 \leq \varepsilon.$$



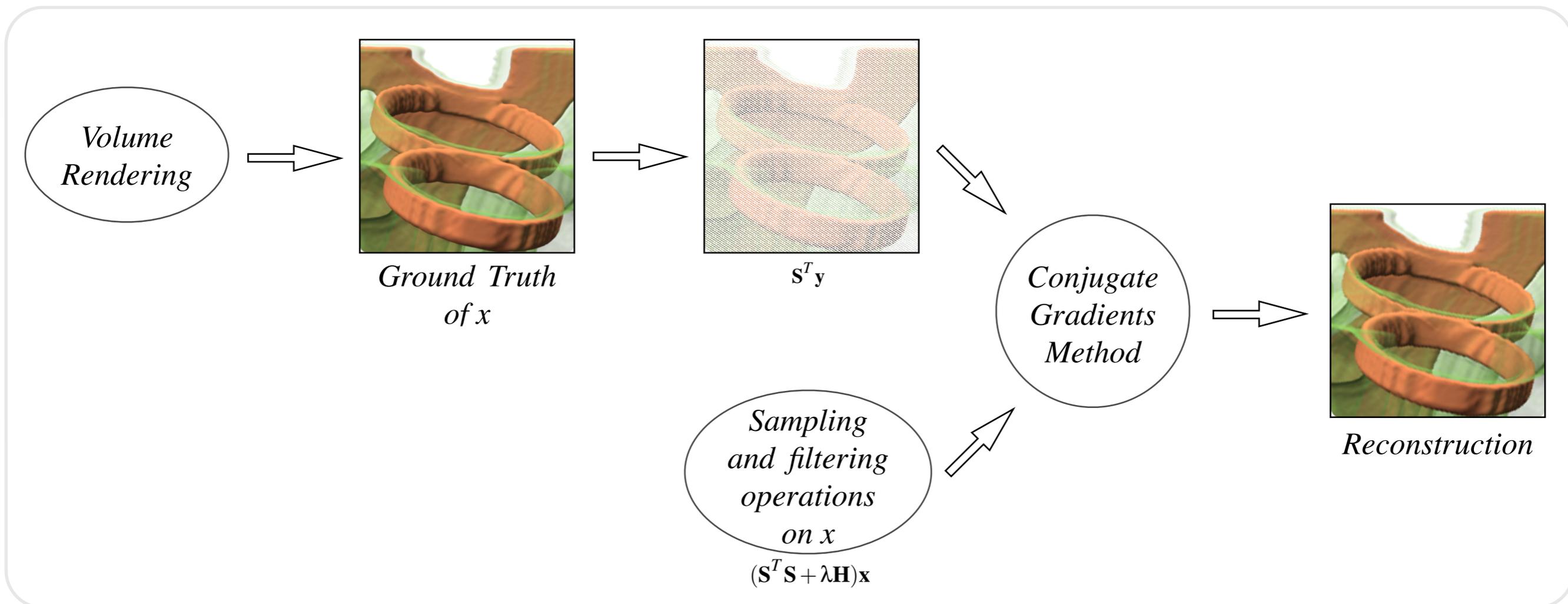
## Total Variation(TV) Minimization

$$\min \|\mathbf{x}\|_{TV} \quad \text{subject to} \quad \|\mathbf{S}\mathbf{x} - \mathbf{y}\|_2 \leq \varepsilon$$



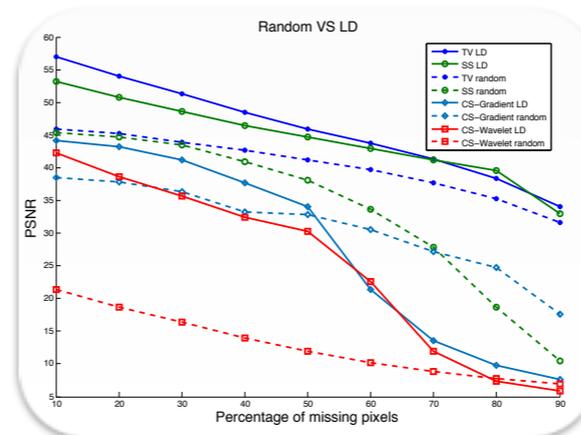
## Smoothness Splines(SS)

$$\min_{\mathbf{x}} \|\mathbf{S}\mathbf{x} - \mathbf{y}\|_2^2 + \lambda \mathbf{x}^T \mathbf{H}\mathbf{x} \quad \longrightarrow \quad (\mathbf{S}^T \mathbf{S} + \lambda \mathbf{H})\mathbf{x} = \mathbf{S}^T \mathbf{y}$$

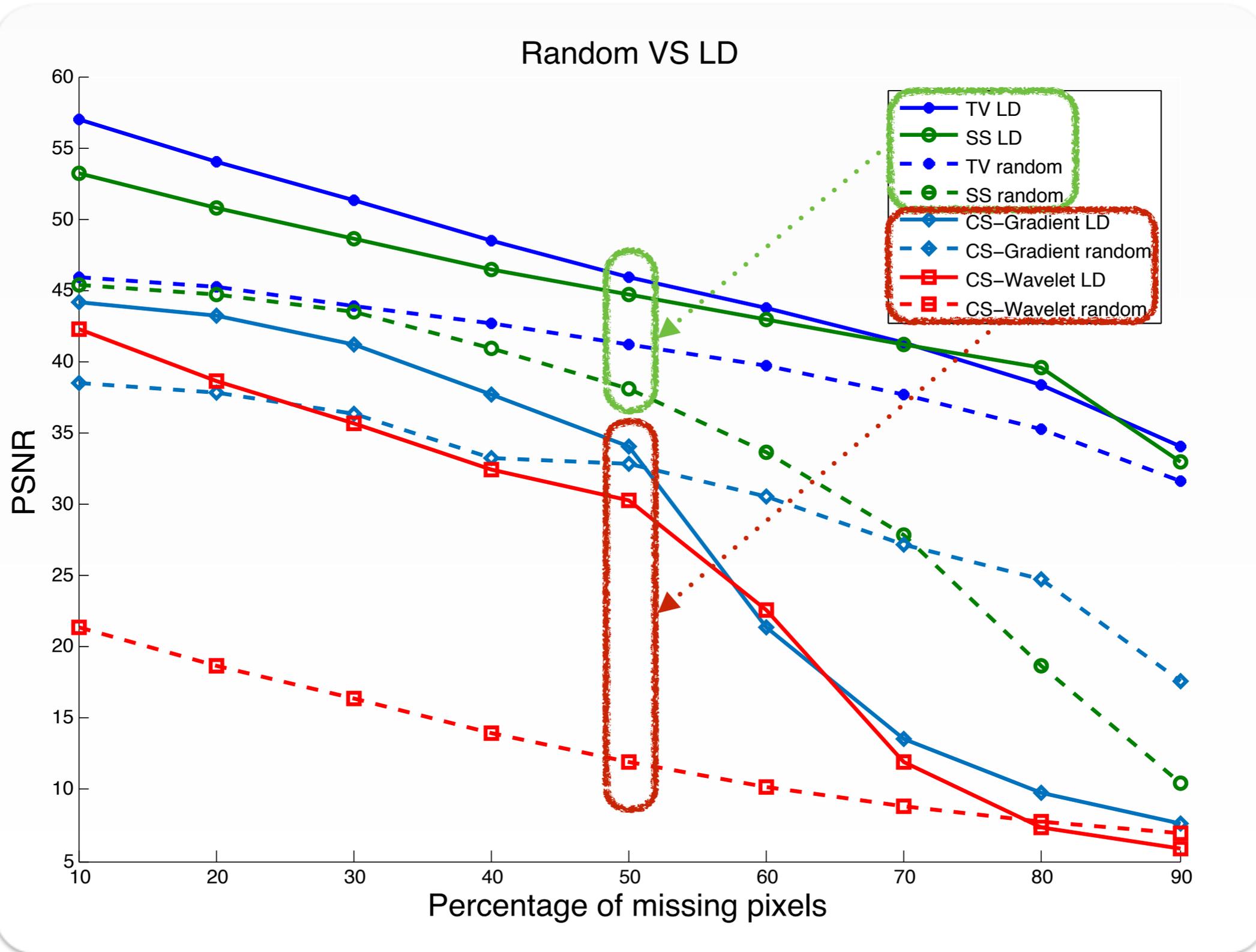


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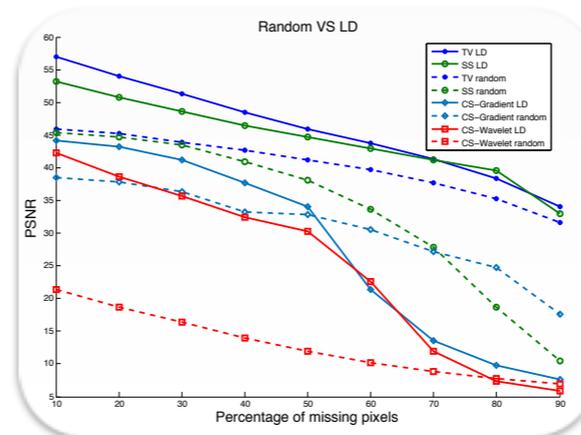
- We recovered the images from **a fraction of the pixels** and experimented with **different percentages of pixels**.



- To measure recovery quality
  - Peak signal-to-noise ratio (
  - Error images**

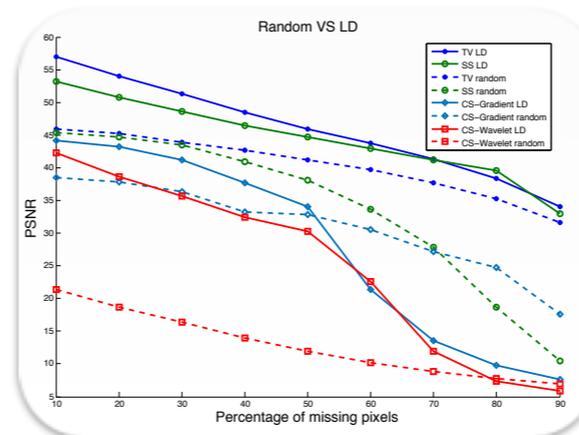


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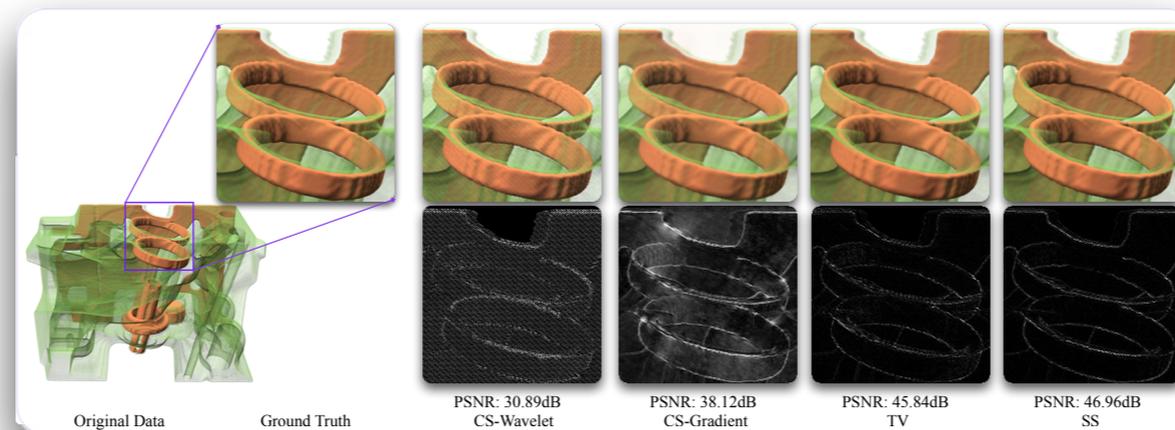


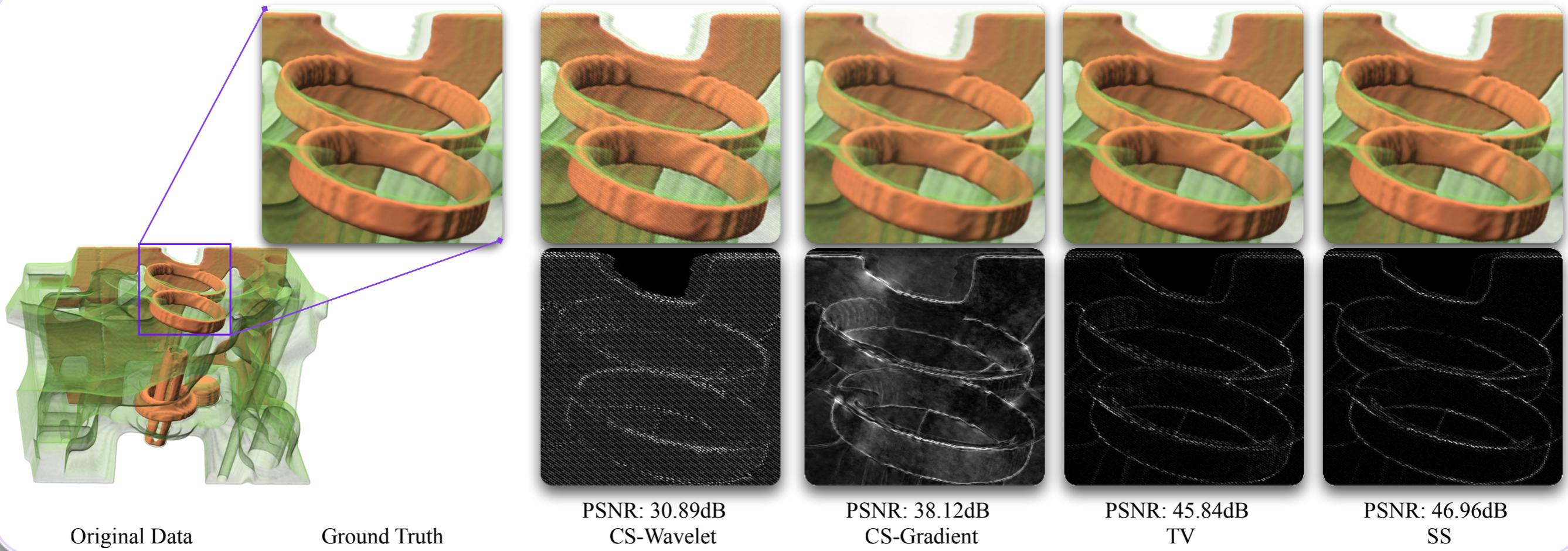
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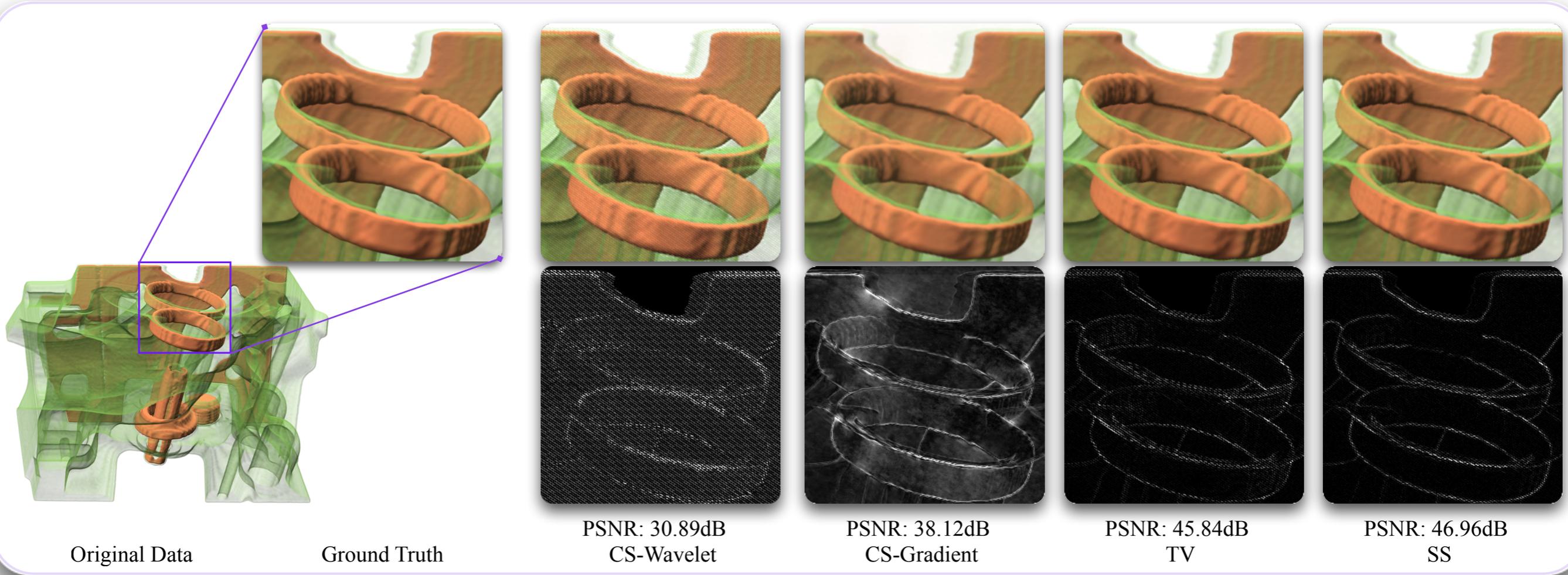
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- To measure recovery quality
  - Peak signal-to-noise ratio (**PSNR**)
  - Error images** in the **CIELUV** colorspace.







Original Data

Ground Truth

PSNR: 30.89dB  
CS-Wavelet

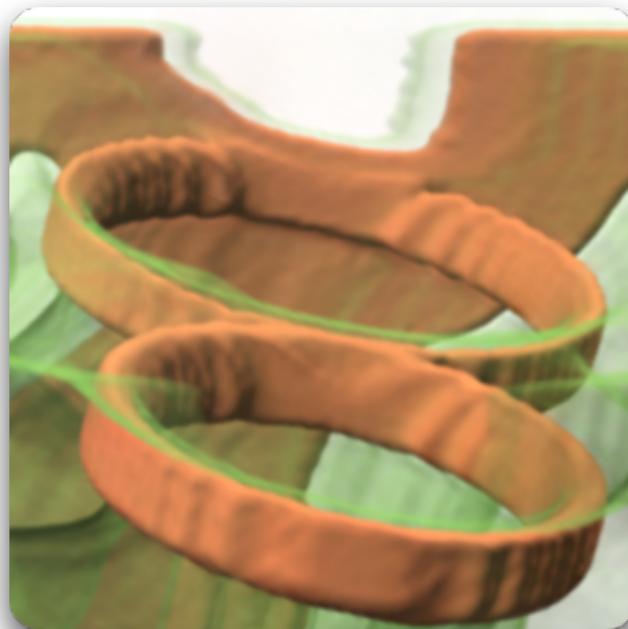
PSNR: 38.12dB  
CS-Gradient

PSNR: 45.84dB  
TV

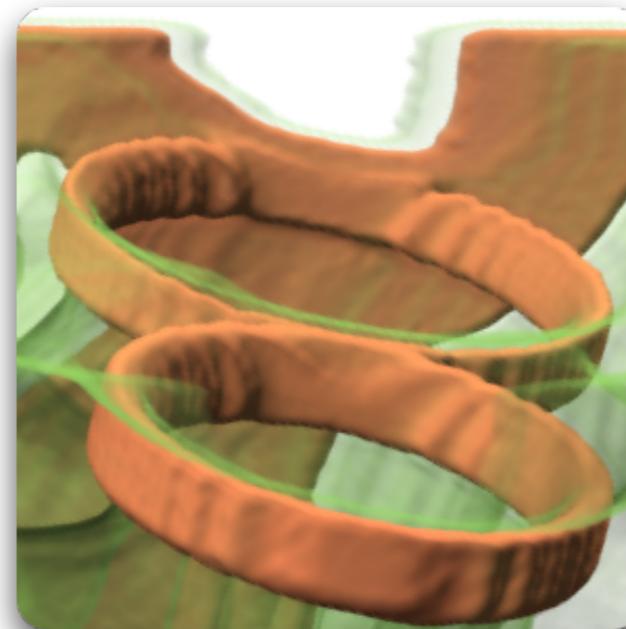
PSNR: 46.96dB  
SS



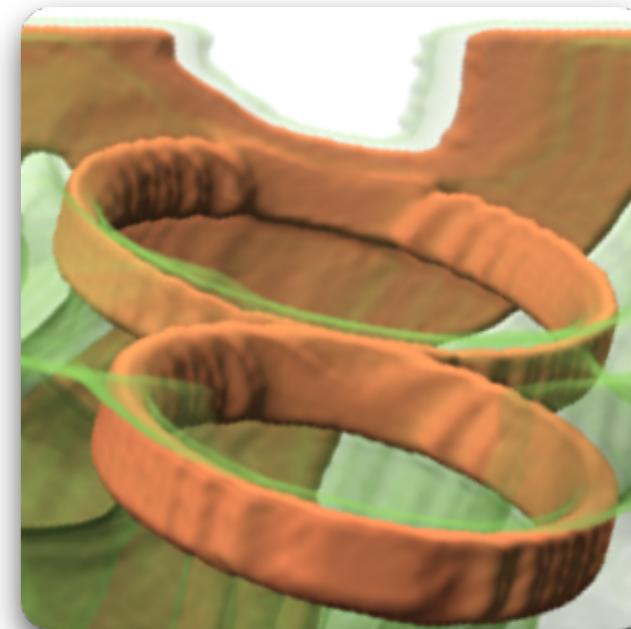
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CS-Wavelet



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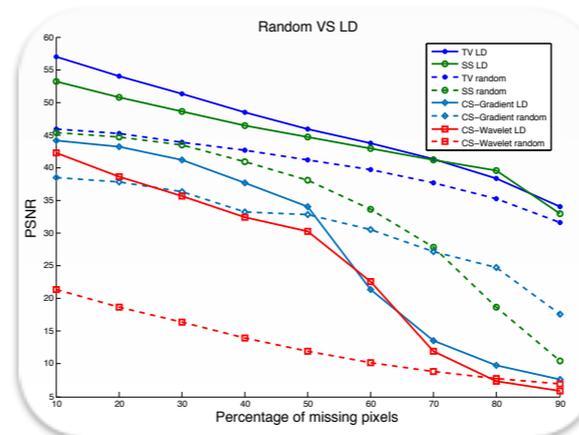


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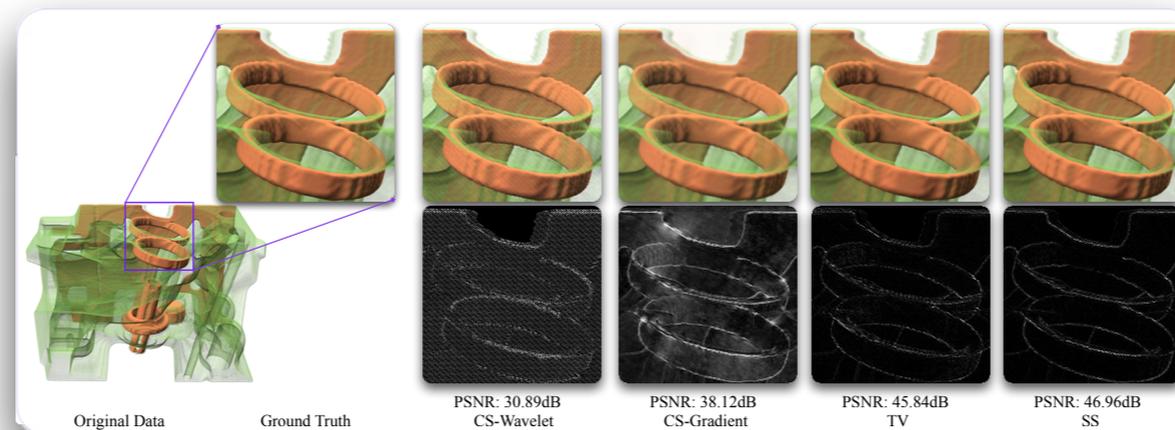


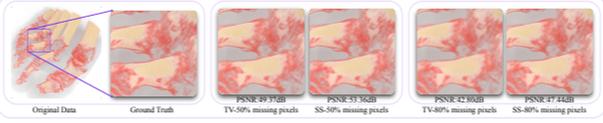
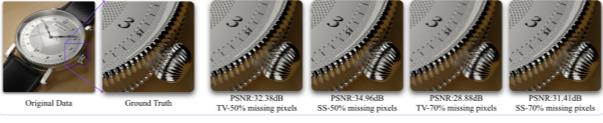
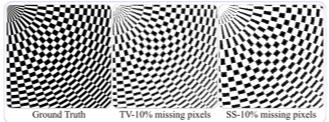
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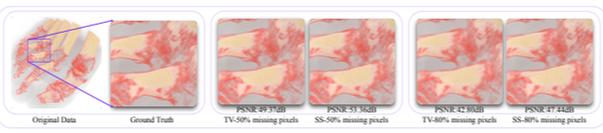
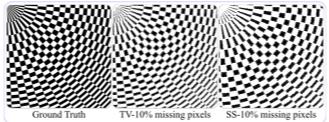
- To measure recovery quality
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Category	Images	TV/SS with LD
Direct Volume Rendering		
Physically-based Rendering		
Iso-Surface Rendering		
Non-Smooth Images		

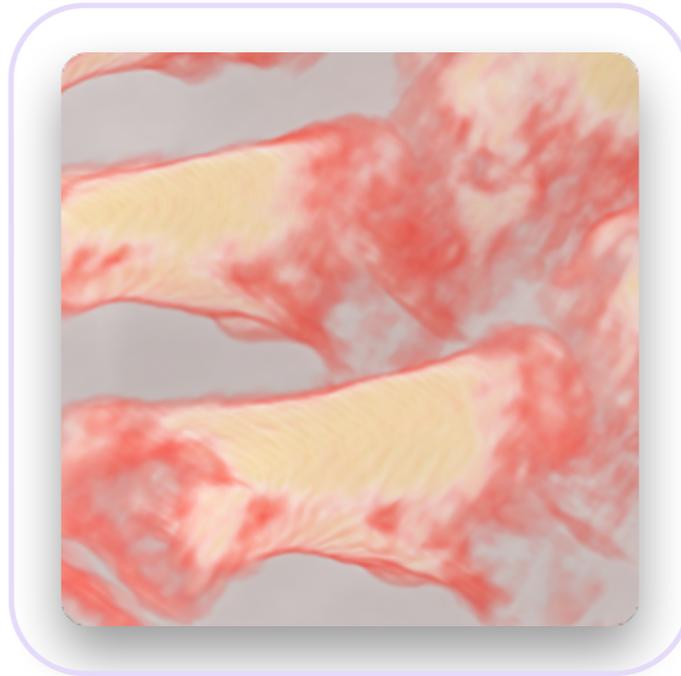
★ ★ ★ — Consistent recovery  
 ★ ★ — Good recovery

★ — Acceptable recovery  
 ☹ — Unacceptable recovery

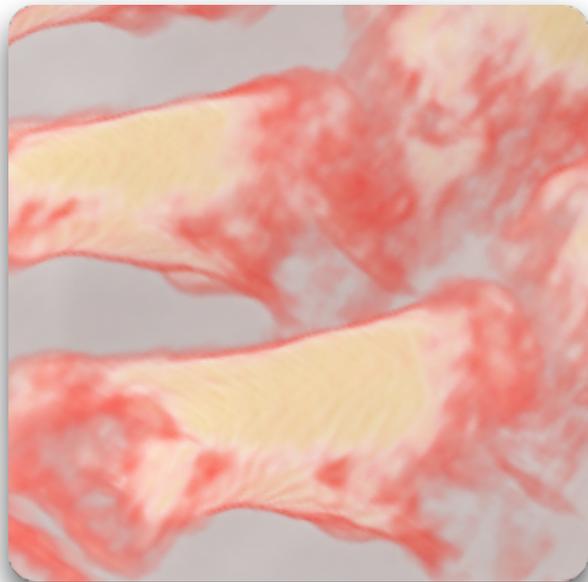
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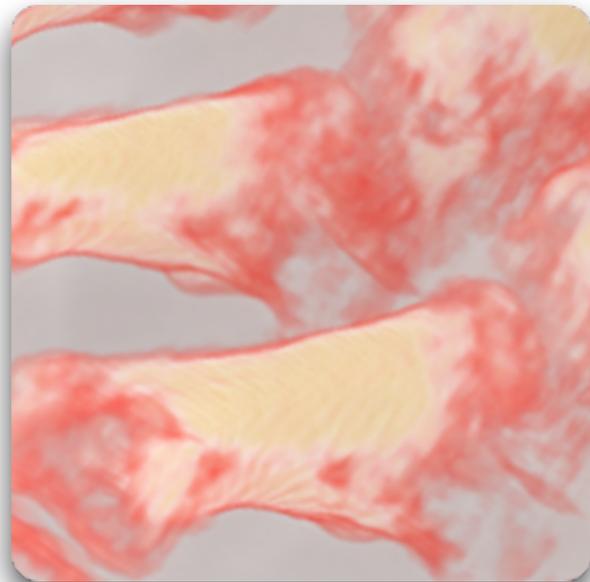


Ground Truth



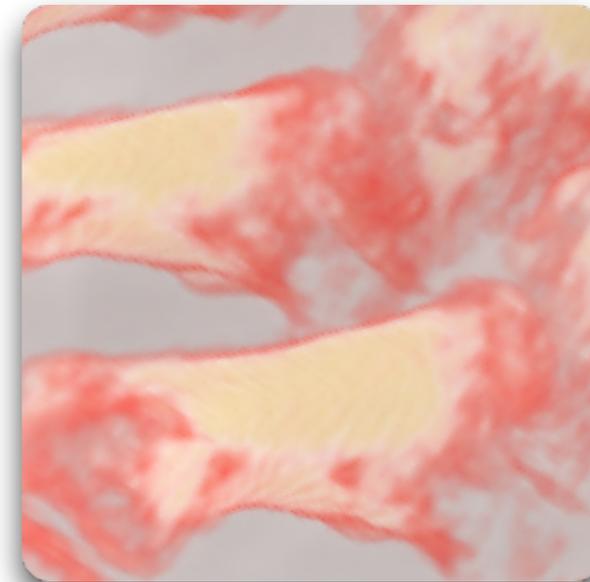
PSNR:49.37dB

TV-50% missing pixels



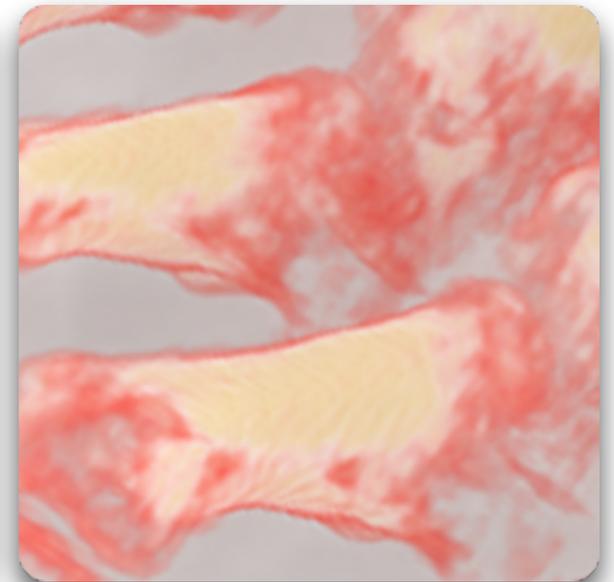
PSNR:53.36dB

SS-50% missing pixels



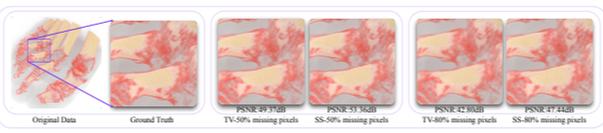
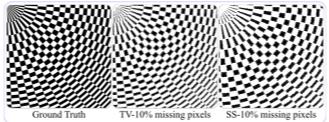
PSNR:42.80dB

TV-80% missing pixels



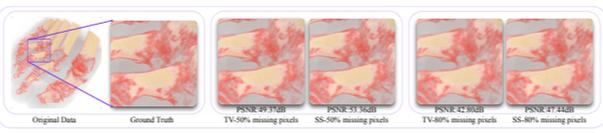
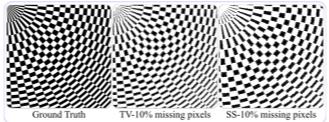
PSNR:47.44dB

SS-80% missing pixels

Category	Images	TV/SS with LD
Direct Volume Rendering		
Physically-based Rendering		
Iso-Surface Rendering		
Non-Smooth Images		

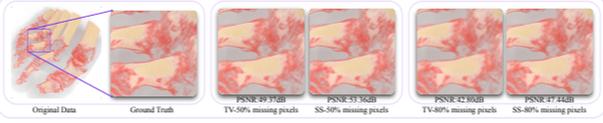
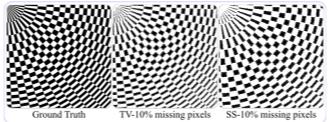
★ ★ ★ — Consistent recovery  
 ★ ★ — Good recovery

★ — Acceptable recovery  
 ☹ — Unacceptable recovery

Category	Images	TV/SS with LD
Direct Volume Rendering		
Physically-based Rendering		
Iso-Surface Rendering		
Non-Smooth Images		

 — Consistent recovery  
 — Good recovery

 — Acceptable recovery  
 — Unacceptable recovery

Category	Images	TV/SS with LD
Direct Volume Rendering		
Physically-based Rendering		
Iso-Surface Rendering		
Non-Smooth Images		

 — Consistent recovery  
 — Good recovery

 — Acceptable recovery  
 — Unacceptable recovery



Ground Truth



PSNR:32.38dB  
TV-50% missing pixels



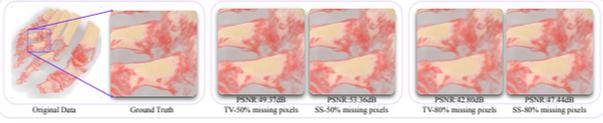
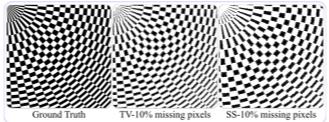
PSNR:34.96dB  
SS-50% missing pixels



PSNR:28.88dB  
TV-70% missing pixels

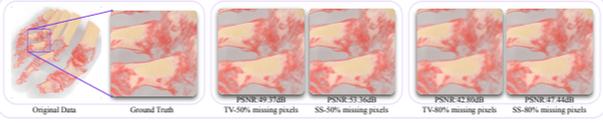
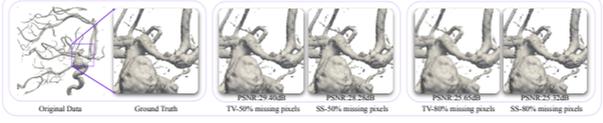
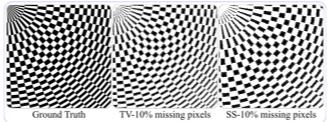


PSNR:31.41dB  
SS-70% missing pixels

Category	Images	TV/SS with LD
Direct Volume Rendering		
Physically-based Rendering		
Iso-Surface Rendering		
Non-Smooth Images		

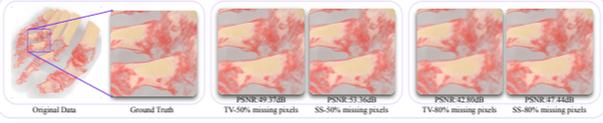
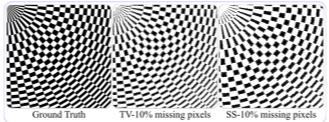
 — Consistent recovery  
 — Good recovery

 — Acceptable recovery  
 — Unacceptable recovery

Category	Images	TV/SS with LD
Direct Volume Rendering		
Physically-based Rendering		
Iso-Surface Rendering		
Non-Smooth Images		

 — Consistent recovery  
 — Good recovery

 — Acceptable recovery  
 — Unacceptable recovery

Category	Images	TV/SS with LD
Direct Volume Rendering		
Physically-based Rendering		
Iso-Surface Rendering		
Non-Smooth Images		

 — Consistent recovery  
 — Good recovery

 — Acceptable recovery  
 — Unacceptable recovery

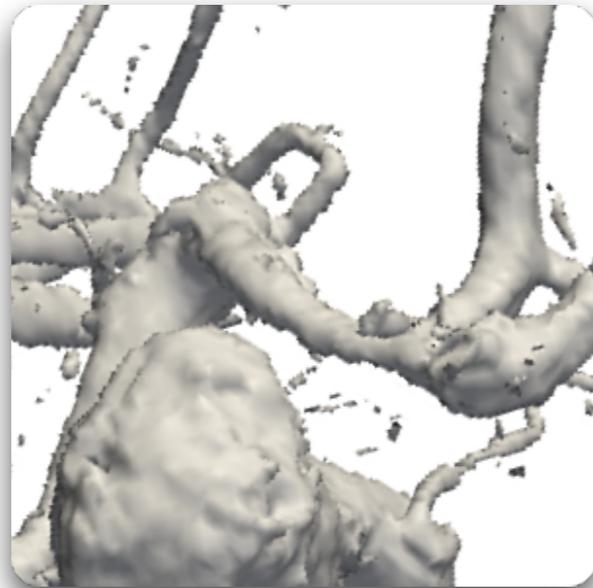


Ground Truth



PSNR:29.40dB

TV-50% missing pixels



PSNR:28.28dB

SS-50% missing pixels



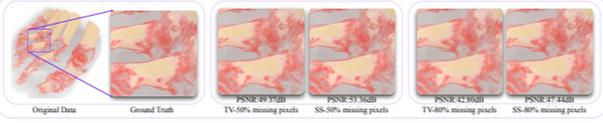
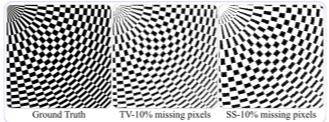
PSNR:25.65dB

TV-80% missing pixels



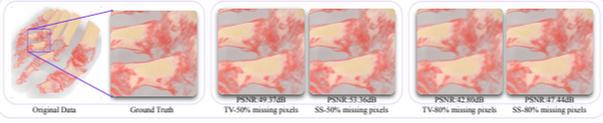
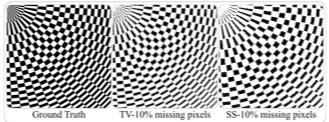
PSNR:25.32dB

SS-80% missing pixels

Category	Images	TV/SS with LD
Direct Volume Rendering		
Physically-based Rendering		
Iso-Surface Rendering		
Non-Smooth Images		

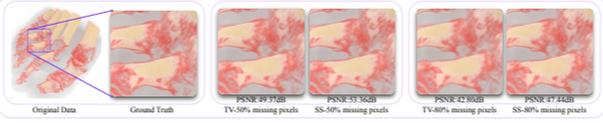
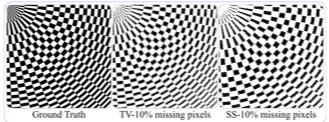
 — Consistent recovery  
 — Good recovery

 — Acceptable recovery  
 — Unacceptable recovery

Category	Images	TV/SS with LD
Direct Volume Rendering		
Physically-based Rendering		
Iso-Surface Rendering		
Non-Smooth Images		

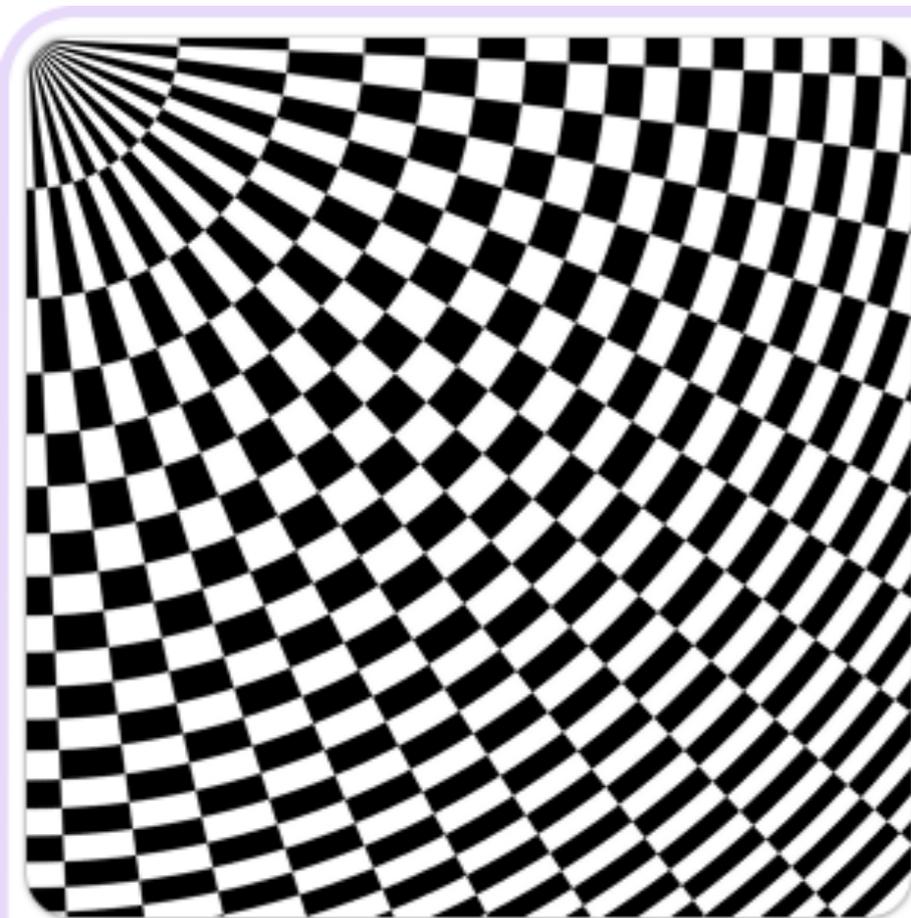
 — Consistent recovery  
 — Good recovery

 — Acceptable recovery  
 — Unacceptable recovery

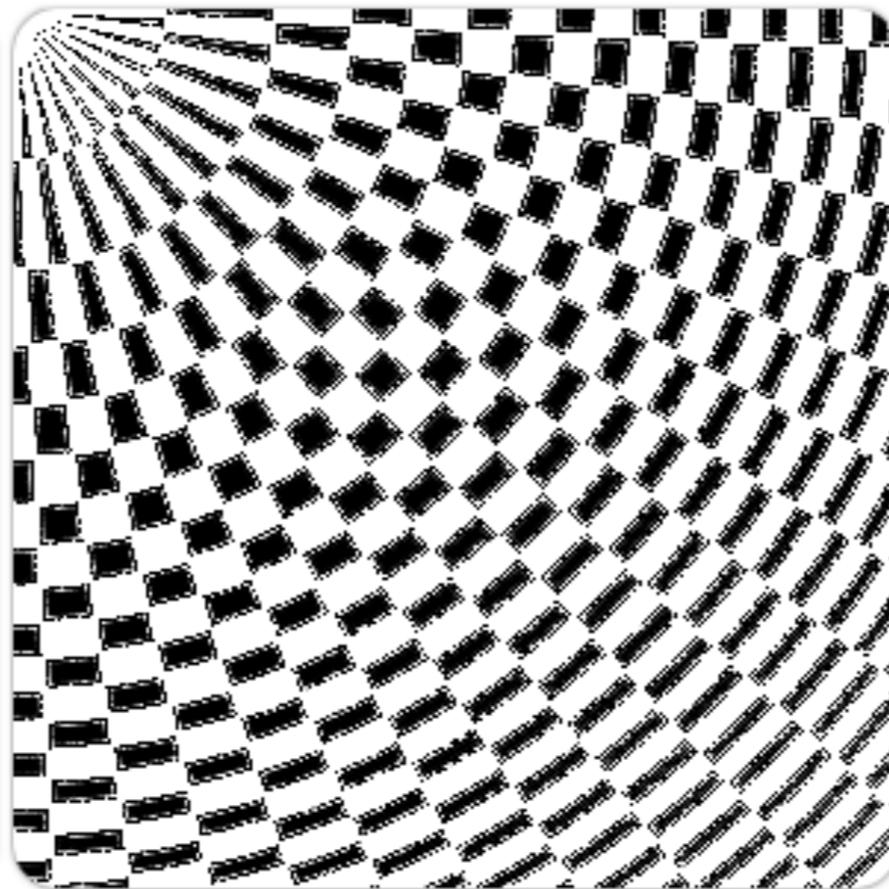
Category	Images	TV/SS with LD
Direct Volume Rendering		
Physically-based Rendering		
Iso-Surface Rendering		
Non-Smooth Images		

 — Consistent recovery  
 — Good recovery

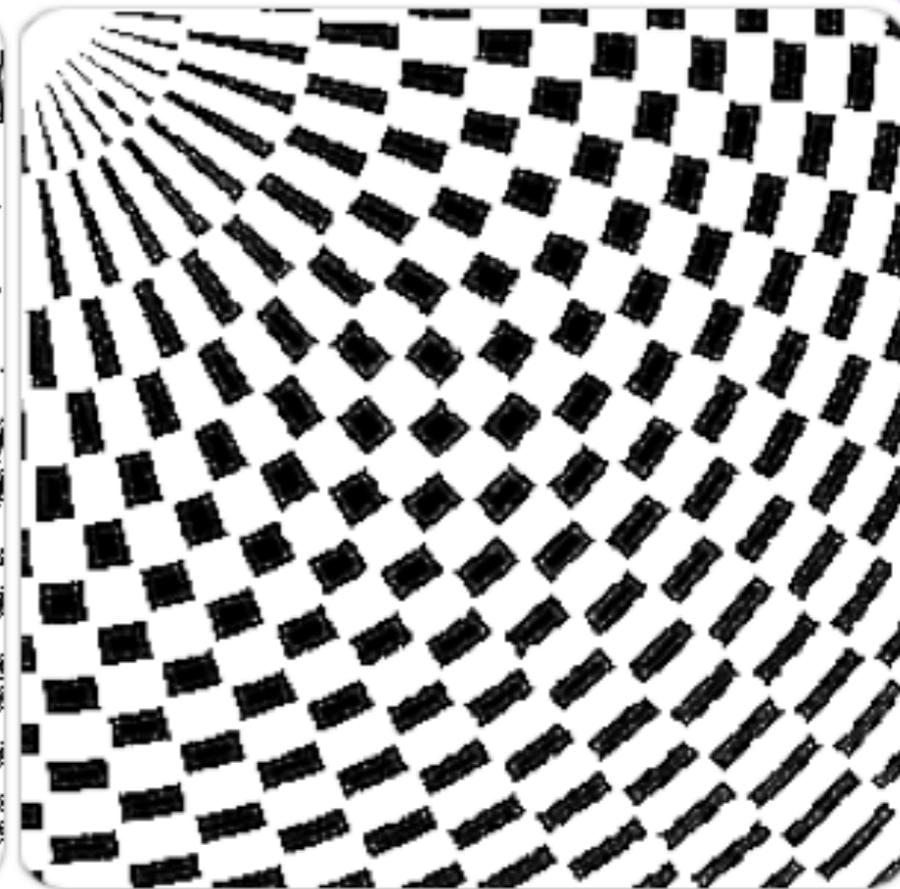
 — Acceptable recovery  
 — Unacceptable recovery



Ground Truth

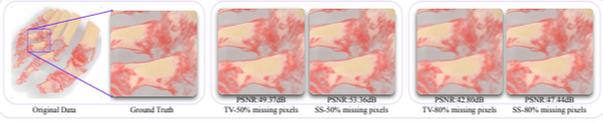
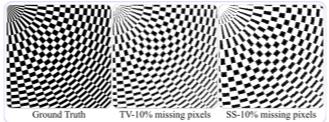


TV-10% missing pixels



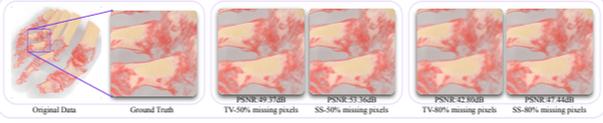
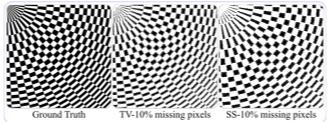
SS-10% missing pixels



Category	Images	TV/SS with LD
Direct Volume Rendering		
Physically-based Rendering		
Iso-Surface Rendering		
Non-Smooth Images		

 — Consistent recovery  
 — Good recovery

 — Acceptable recovery  
 — Unacceptable recovery

Category	Images	TV/SS with LD
Direct Volume Rendering		
Physically-based Rendering		
Iso-Surface Rendering		
Non-Smooth Images		

 — Consistent recovery  
 — Good recovery

 — Acceptable recovery  
 — Unacceptable recovery

1. Motivation
2. Research Question
3. Methodologies
4. Results
- 5. Conclusion**

- We presented three different methods for recovering images from a subset of the pixels
- CS-based approaches are not suitable for this problem as we are restricted to making pixel measurements
- Answer the question of which method is the most suitable for compressive volume rendering via a very small fraction of rendered pixels.

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# Thank you!



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<http://visagg.cpsc.ucalgary.ca>

<b>Resolution</b>	1200 * 1200	600 * 600	900 * 900
<b>Rendering Time</b>	1	0.25	0.56
<b>CS-Wavelet</b>	0.024	<ul style="list-style-type: none"> <li>• Rendering time for 1200 * 1200 image is the baseline</li> <li>• Relative recovery time for different methods is compared against each other</li> </ul>	
<b>CS-Gradient</b>	0.011		
<b>TV</b>	0.011		
<b>SS</b>	0.003		