Icosahedral Maps for a Multiresolution Representation of Earth Data

Mohammad Imrul Jubair\textsuperscript{1}, Usman Alim\textsuperscript{1}, Niklas Röber\textsuperscript{2}, John Clyne\textsuperscript{3} and Ali Mahdavi-Amiri\textsuperscript{1}

\textsuperscript{1}University of Calgary, \textsuperscript{2}German Climate Computing Centre, \textsuperscript{3}National Center for Atmospheric Research
INTRODUCTION
Icosahedral Non-hydrostatic (ICON):

- A 3D Earth model used for numerical weather prediction.
- Earth surface is described and data is assigned.

Jointly developed by the Max Planck Institute for Meteorology (MPI-M) and the German Weather Service (DWD).
ICON is designed via **Discrete Global Grid System (DGGS)**.
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For improved numerical solution, different data is assigned at different locations of the **primal cell (triangle)**.
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- **Vertices**: 2
- **Centroids of hexagons**: (dual of primal cell)
For improved numerical solution, different data is assigned at different locations of the **primal cell (triangle)**.
RESEARCH GOAL
Research Problem

Visualization

- Interactive exploration of ICON demands efficient use of-
  - Memory.
  - I/O bandwidth.

- But, ICON data is high resolution.

*How can we improve visualization?*

One solution is:

- A multiresolution representation of ICON.
  - Level-of-Detail Rendering.
Data Structure

- ICON is represented as *Polygon Soup (face list).*
  - No explicit neighborhood information.

An Example:
Multiresolution needs neighbor vertices to perform its operations, e.g. -
- Convolution
- Downsampling and upsampling

But, *soup* makes it expensive.
- Retrieving neighbors from the soup for every vertex.

*How can we make these operations faster?*
Mapping **unstructured soup** into a **structured 2D representation**, we call it ‘Icosahedral maps’.
Mapping works for all three cell-types.

- Centroids of Hexagons
- Centroids of Triangles
- Centroids of Quads

structured 2D array
LoD representation of Earth data by applying a *multiresolution scheme* on the icosahedral maps.
METHODOLOGY

Icosahedral Maps
Given one **triangle** from a triangle **soup**, a hexagonal fan finds its hexagonal **neighbors** of a vertex and store the information in a **2D array**.
An example:

Triangle soup

array

i  i+1  i+2

j  j+1  j+2
• An example:

Triangle soup

T_1

array
An example:
An example:

```
   T1   T2   Triangle soup

   i    i+1  i+2

   j    j+1  j+2

   array
```
An example:

Hexagonal Fan

Triangle soup

T₁

T₂

array
- **An example:**

```
\begin{array}{ccc}
T_3 & T_1 & T_2 \\
\end{array}
```

Triangle soup

```
\begin{array}{ccc}
\text{Array} \\
T_1 & T_2 \\
\end{array}
```
An example:

Triangle soup

array
- An example:
This traversal scheme will be used for capturing information for three-types of cell centroids of ICON.
Mapping Centroids of Hexagons:

- Vertices of the primal cells

2D array

EO vertex

3D Earth
1. Mapping Centroids of Hexagons:
   - Initial triangle vertices.

2D array

3D Earth

EO vertex
Mapping Centroids of Hexagons:

- Hexagonal fan sweeps to fill up array.

3D Earth
Mapping Centroids of Hexagons:

- One array extract one diamond on Earth.

2D array

3D Earth

Vertex information (lon, lat)

EO vertex

diamond
1. Mapping Centroids of Hexagons:
   - Associated data is also extracted.

2D array

Scalar data

3D Earth
Mapping Technique

2 Mapping Centroids of Quads:
   - Edge midpoints of the primal cells
2. Mapping Centroids of Quads:
   - Edge midpoints of the primal cells
Mapping Technique

2. Mapping Centroids of Quads:
   - Vertices stored as quad soup
Mapping Centroids of Quads:

- Three directional edges
Mapping Technique

2 Mapping Centroids of Quads:
   - Three directional edges
Mapping Centroids of Quads:

- Three directional edges
Mapping Centroids of Quads:
  — Modified hexagonal fan.
2 Mapping Centroids of Quads:

- Modified hexagonal fan.
- Example:
Mapping Technique

2. Mapping Centroids of Quads:
   - Modified hexagonal fan.
   - *Example:*
2. Mapping Centroids of Quads:
   - Modified hexagonal fan.
   - Example:

   ![Diagram of quad soup]

   quad soup
Mapping Centroids of Quads:

- Modified hexagonal fan.
- Example:
Mapping Technique

2. Mapping Centroids of Quads:
   - Stored in array.

![2D array diagram]

- $e_1$
- $e_2$

2D array
2. Mapping Centroids of Quads:
   - Stored in array.
Mapping Centroids of Quads:

- At polar vertex:
2 Mapping Centroids of Quads:
   — Need to access adjacent diamond.
Mapping Centroids of Quads:

- Minor irregularity at border.
- Topology is preserved.
Mapping Technique

2. Mapping Centroids of Quads:
   - Along other directions:
3 Mapping Centroids of Triangle:
   – The primal cells.
Mapping Centroids of Triangle:

- Connecting vertices with centroids.
- Splitting into triangle.

new triangle
soup
Mapping Centroids of Triangle:

- Data at the vertices are assigned.
3 Mapping Centroids of Triangle:

- Fan sweeping:
3. Mapping Centroids of Triangle:

- Fan sweeping:
Mapping Centroids of Triangle:

- Array has null entries.
SUMMARY OF ICOSAHEDRAL MAPS
Mapping to a 2D structure
Mapping technique extract a diamond.
For entire Earth: total 10 diamonds.
For entire Earth: total 10 diamonds. [for every cell-types]
METHODOLOGY

Multiresolution
Discrete Hexagonal Wavelet

- **Hexagonal Wavelet bases** [Cohen & Schlenker ’93]

  A linear approximation of the data is obtained by linear box spline.

\[
f(x) = \sum_k F[k] \varphi(x - Lk)
\]

- Coarse-to-fine reconstruction:

\[
f(x) = \sum_k C[k] \varphi(x/2 - Lk) + \sum_{i=1}^3 \sum_k D_i[k] \psi_i(x/2 - Lk).
\]
- Sub-band coding scheme:
Sub-band coding scheme:

Operations:

Convolution with filters

up/downsampling
MR on icosahedral maps:
MR on icosahedral maps:

- **Fine**
- **Coarse**
- **Details**
- **Reconstructed**
Compression

- Removing details with less energy
- Quantile threshold:
  - Choose a threshold on frequency distribution.
Compression

- Removing details with less energy
- Quantile threshold:
  - Choose a threshold on frequency distribution.
  - Keep the top percentages.

Threshold = 90%
RESULTS
Hexagonal cells

where data is at the centroids of hexagons
where data is at the vertices of triangles
Visual Results: ICON Data

MR on ICOSAHEDRAL MAPS

Coarse Level
Visual Results: ICON Data

Reconstructed (Threshold = 90%)
### Other cells:

<table>
<thead>
<tr>
<th>Input cell</th>
<th>Icosahedral Maps: Triangle grid</th>
<th>MR: Coarse level</th>
<th>MR: Reconstructed (threshold = 90%)</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1.png" alt="Centroids of Triangles" /></td>
<td><img src="image2.png" alt="Centroids of Triangles" /></td>
<td><img src="image3.png" alt="Centroids of Triangles" /></td>
<td><img src="image4.png" alt="Centroids of Triangles" /></td>
</tr>
<tr>
<td><img src="image5.png" alt="Centroids of Quads" /></td>
<td><img src="image6.png" alt="Centroids of Quads" /></td>
<td><img src="image7.png" alt="Centroids of Quads" /></td>
<td><img src="image8.png" alt="Centroids of Quads" /></td>
</tr>
</tbody>
</table>
Visual Results: Synthetic Data

Centroids of hexagons

Centroids of Triangles

Centroids of Quads

Fine

Reconstructed
(threshold = 90%)
- We focused only on the centroids of the primal cells (triangles).

- **Challenges:**
  - number of EO vertices is not 12.
  - no stopping conditions for fan sweeping.
ICON Data (slice)

Hurricane data
ICON Data (slice)

Hurricane data
ICON Data (slice)

Fine

Coarse (LoD-1)

Coarse (LoD-2)
ICON Data (slice)

Fine

Reconstructed

Reconstructed (90%)
- **Quality vs. Compression**: (ICON Data)
### Quantitative Results

- **Quality vs. Compression:** (Synthetic Data)

![Graph showing Quality vs. Compression](image-url)

- Quality vs. Compression (threshold)
- Quality (PSNR) vs. Compression (threshold)

Key Points:
- Different data types (Hexagons, Quads, Triangles) are compared.
- The graph illustrates the trade-off between Quality (PSNR) and Compression (threshold) for each data type.
- KD – tree.
- KNN – search.
- 1-to-3 refinement parallel.
- Hexagonal fan searches locally.
CONCLUSION
Icosahedral Maps:
Maps connectivity information for all three cell-types into a structured grid representation.
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Maps connectivity information for all three cell-types into to a structured grid representation.

- Neighboring information is easy to access simply using indexing. Operations involved in MR is straightforward.
  - Convolution
  - Downsample and Upsample

- GPU friendly because our 2D representation -
  - is easily fit into the GPU using textures.
  - allows to use barycentric interpolation for all types of data.
Level-of-Detail representation of Earth data:

- Applying a hexagonal wavelet scheme on the icosahedral maps to render scalar data at a coarser resolution.
  - save time

- compression via thresholding.
  - save space

Rendered at fine level

Rendered at coarse (2\textsuperscript{nd} level-of-detail)
• GPU Implementation.

• Mapping the whole slice into one single array.
  o This can be done by taking advantage of hexagonal fan traversal.
• Our pipeline:
Conclusion

• Question:

- Why does ICON come with soup structure?
- Is 2D structure possible? Any problem for simulation?
THANKS
QUESTION?
MORE RESULTS
variable: rho

Fine

LoD-1

LoD-2
variable: temp

Fine

LoD-1

LoD-2
variable: pres

Fine

LoD-1

LoD-2
variable: 1

Fine

LoD-1

LoD-2
variable: 6

Fine

LoD-1

LoD-2