OPERATOR DISCRETIZATION IN SHIFT-INVARIANT SPACES

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A *Shift-invariant* space — spanned by the lattice translates of an admissible generating function — is typically used to approximate a function from measurements that lie at the nodes of a lattice. Inspired by the univariate treatment of Blu and Unser [1], we perform a multivariate error analysis of interpolative and quasi-interpolative techniques. This yields a Fourier domain error kernel that can be used to compare the asymptotic performance of different lattice-generator combinations when approximating the same function f. We then analyze the problem of approximating, from the measurements of f, the scalar field Lf, where L is a differential operator. We focus on approximation techniques that can be expressed as a discrete lattice-based convolution of the measurements with a suitable discretization of L. Our main result is a modified Fourier error kernel that depends on the Fourier transform of L and the discrete-time Fourier transform of its discretization. We examine two cases in detail: the accurate estimation of the gradient of a function [2, 3], and the approximate solution to Poisson's equation within a rectangular domain with homogeneous Dirichlet boundary conditions [4]. In either case, we follow a two-stage recipe that yields operator discretizations that are asymptotically optimal. In the first stage, a higher order approximation $\tilde{f}$ of f is sought in an auxiliary space. Then, the quantity $L\tilde{f}$ is orthogonally projected to the target space to yield the desired approximation. Our approximation methodologies are validated by conducting quantitative and qualitative experiments on the three-dimensional Cartesian and body-centered cubic lattices.

REFERENCES

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